INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, VOL. 37, 645–667 (1994)

ON STABILITY AND CONVERGENCE OF FINITE ELEMENT APPROXIMATIONS OF BIOT'S CONSOLIDATION PROBLEM

MÁRCIO A. MURAD AND ABIMAEL F. D. LOULA

Laboratório Nacional de Computação Científica - LNCC/CNPq Rua Lauro Muller 455, 22290—Rio de Janeiro, Brazil

SUMMARY

Stability and convergence analysis of finite element approximations of Biot's equations governing quasi-static consolidation of saturated porous media are discussed. A family of decay functions, parametrized by the number of time steps, is derived for the fully discrete backward Euler-Galerkin formulation, showing that the pore-pressure oscillations, arising from an unstable approximation of the incompressibility constraint on the initial condition, decay in time. Error estimates holding over the unbounded time domain for both semidiscrete and fully discrete formulations are presented, and a post-processing technique is employed to improve the pore-pressure accuracy.

1. INTRODUCTION

Variational principles and finite element approximations for Biot's quasi-static consolidation theory [1–4] have been proposed by many investigators [5–11] from the view point of applications to Geomechanics. To solve this problem, time-stepping integration schemes, whose stability and accuracy are discussed in References 12–14, are usually employed, combined with finite element approximation in the space domain. Galerkin finite element approximations of Biot's equations in terms of soil displacement and pore pressure have presented spurious oscillations in the pressure field in the early stage of the consolidation process[15–17] for some combinations of displacement and pore-pressure finite element spaces, specially those with equal order of interpolation. To avoid this misbehaviour, different orders of interpolation, usually one order lower for pore pressure compared to displacements or reduced integration techniques,[16] have been adopted. This lack of stability close to the initial-time results from an unstable approximation of the initial condition. At $t = 0$, we shall solve an incompressible elasticity problem (same structure of Stokes flow) in displacement and pore-pressure formulation whose finite element approximation must satisfy the Babuška–Brezzi condition (see, e.g. References 18–21).

The analysis of the influence of error in the initial data over stable and unstable approxima-