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Higher-order gradient post-processings for second-order elliptic problems

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Abstract

Global, element-by-element and macroelement post-processing recovery techniques based on least-square residuals of equilibrium equation and irrotationality condition are proposed for second-order elliptic problems. Improved accuracy for the flux finite element approximations is obtained with low computational cost and easy implementation. Error estimates are derived and numerical experiments are reported confirming the higher-order rates of convergence predicted in the analysis.

1. Introduction

Finite element approximations of second-order elliptic problems are usually constructed based on a single field formulation or displacement method with fluxes or stresses calculated by differentiating the finite element solution and using the appropriate constitutive equations. This obvious approach, however, leads to lower-order approximations for fluxes and stresses compared to the primal variable, and in addition, the corresponding balance equation is enforced in an extremely weak sense. In most applications, as for example in heat conduction, percolation in porous media, or stress analysis, fluxes or stresses are the variables of main interest. Thus, alternative formulations have been developed aimed at improving their approximations. The first alternative to displacement methods would be equilibrium formulations in which flux becomes the primitive variables [1]. This approach often leads to a non-standard implementation for requiring construction of finite element spaces incorporating the equilibrium constraint. Alternatively, dual mixed methods, based on the simultaneous approximation of potential and flux [3], have been successfully used in engineering applications [4, 5].

For unconstrained problems, mixed methods may not present significative advantages over displacement methods for requiring higher number of degrees of freedom and for leading to indefinite systems. Great improvement in the solution of RT and BDM mixed systems is obtained with a hybrid implementation [6] where the continuity of the normal component of the flux is enforced by Lagrange multipliers defined on the interelement boundaries. Another characteristic of mixed methods is the required compatibility condition between the finite element spaces to ensure existence and uniqueness of solution (LBB condition [7]) which reduces the flexibility in the choice of stable finite element spaces. For example, an appropriate analogue of the mixed formulation proposed by Raviart–Thomas and Brezzi–Douglas–Marini is not immediately extended to Hellinger–Reissner formulations of

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