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A Multi-Scale Theory of Swelling Porous Media: I. Application to One-Dimensional Consolidation

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Abstract. A theory is developed which describes flow in multi-scale, saturated swelling media. To upscale information, both the hybrid theory of mixtures and the homogenization technique are employed. In particular, a model is formulated in which vicinal water (water adsorbed to the solid phase) is treated as a separate phase from bulk (non-vicinal) water. A new form of Darcy's law governing the flow of both vicinal and bulk water is derived which involves an interaction potential to account for the swelling nature of the system. The theory is applied to the classical one-dimensional consolidation problem of Terzaghi and to verify Low's empirical, exponential, swelling result for clay at the macroscale.

Key words: Swelling clay soil, multi-scale flow, hybrid mixture theory, homogenization, consolidation.

1. Introduction

The development of theories to simultaneously represent consolidation and the flow of water in saturated clay soils began with Terzaghi [48] and Biot [11]. Essentially, Biot and Terzaghi developed linear poroelastic models based upon a phenomenological approach at the macroscale. Their models are now well established and Biot's model can be recovered via a linearization procedure within a more general thermomechanical framework of modern mixture theory (e.g. Crochet and Naghdi [20], Green and Steel [31]) or by applying a homogenization technique to a local pore-scale problem where the solid is considered to be linearly elastic and the fluid is assumed to be Stokesian (Auriault [4]). Although limited to the linear elastic range, the Biot and Terzaghi models have been the basis for the development of more general approaches aimed at removing some of their restrictive assumptions. For one-dimensional consolidation, nonlinear extensions of the Terzaghi theory have been proposed by using nonlinear stress–strain relationships (e.g. Davis and

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