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A new multiscale scheme for computing statistical moments in single phase flow in heterogeneous porous media

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ABSTRACT

In this work we develop a new multiscale procedure to compute numerically the statistical moments of the stochastic variables which govern single phase flow in heterogeneous porous media. The technique explores the properties of the log-normally distributed hydraulic conductivity, characterized by power-law or exponential covariances, which shows invariance in its statistical structure upon a simultaneous change of the scale of observation and strength of heterogeneity. We construct a family of equivalent stochastic hydrodynamic variables satisfying the same flow equations at different scales and strengths of heterogeneity or correlation lengths. Within the new procedure the governing equations are solved in a scaled geology and the numerical results are mapped onto the original medium at coarser scales by a straightforward rescaling. The new procedure is implemented numerically within the Monte Carlo algorithm and also in conjunction with the discretization of the low-order effective equations derived from perturbation analysis. Numerical results obtained by the finite element method show the accuracy of the new procedure to approximated the two first moments of the pressure and velocity along with its potential in reducing drastically the computational cost involved in the numerical modeling of both power-law and exponential covariance functions.

1. Introduction

During the last two decades stochastic modeling of flow in porous media has been successfully applied in the quantification of uncertainty and became one of the main subjects in subsurface hydrology and reservoir modeling (see e.g. [27,48,42]). In the usual mathematical framework seated in the geostatistical working hypothesis the formation properties are conveniently treated as random space functions whose statistics are usually inferred from few experimental data. The randomness involved in the conductivity induces fluctuations in the seepage velocity through Darcy's law and consequently leads to hydrodynamics in random media governed by stochastic partial differential equations (see e.g. [26]). One of the primary challenges in the modeling of such complex phenomena is the reliable computation of the statistical moments of the random variables. Such task has to be accomplished in an accurate and computationally tractable manner, particularly when dealing with large-scale problems [15,30,34,44,14,17].

A variety of stochastic approaches has been developed aiming at capturing the large-scale effects of variability upon the hydrodynamics. The most conceptually straightforward method to solve stochastic equations is the Monte Carlo algorithm in which the relevant statistical moments are determined by averaging an ensemble of equally probable deterministic solutions, each one obtained from a single highly resolved realization of the fine grid heterogeneity information of the input property. Despite the Monte Carlo method is appealing due to its simplicity and has been widely applied in the numerical solution of a broad range of linear and nonlinear stochastic partial differential equations, it suffers from the drawbacks associated with the high computational costs involved in its numerical implementation. In addition to the large number of realizations often required to achieve statistical convergence, the high computational effort arises from the necessity of resolving numerically the high frequency fluctuations of the random variables which often requires fine grid resolution scales. This latter constraint enforces an upper bound on the size of the finite element mesh which is restricted to the same order of the scale of the heterogeneity to avoid loss of regularity of the solution when adopting coarser meshes.

An alternative family of methods to treat stochastic partial differential equations governing flow in random media with mild heterogeneity is the perturbation based approaches commonly

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