Errata for “Algebraic methods to compute Mathieu functions” by D Frenkel and R Portugal, J. Phys. A: Math. Gen. 34(2001)

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p. 3543 equations (6a) and (6b): The denominator is $i(2r+i)$, not $4i(r+i)$. Same in equations (14a) and (14b) on page 3544.

p. 3549 line 2: We think the normalization value is 2π also for $q \neq 0$ and $m = 0$, for the formula given in (44). At least this is true for the corrected formula given below.

p. 3549 equation (44): The correct formula is

$$\begin{aligned} \frac{1}{C_r^2} &= 1 + \frac{1}{1+\delta_{r,0}} \sum_{k=1}^{\infty} \sum_{l=1}^{k-1} \sum_{i=-\min(k-l,l,\lfloor r/2 \rfloor)}^{\min(k-l,l)} \frac{1}{1+\delta_{2i,-r}} \\ &\quad \times \left(A_{2i}^{k-l} + A_{-2i-2r}^{k-l}\right) \left(A_{2i}^{l} + A_{-2i-2r}^{l}\right) q^k \end{aligned}$$

Proof: The L_2-norm of the Fourier series (16a) is

$$\begin{aligned} & \int_0^{2\pi} \left(\frac{De_0^{(r)}}{2} + \sum_{m\geq 1} De_m^{(r)} \cos(mz) \right)^2 dz \\ = & \int_0^{2\pi} \left(\frac{De_0^{(r)}}{2} \right)^2 dz + \sum_{m\geq 1} \int_0^{2\pi} \left(De_m^{(r)} \cos(mz) \right)^2 dz \\ = & \; 2\pi \left(\frac{De_0^{(r)}}{2} \right)^2 + \sum_{m\geq 1} \pi \left(De_m^{(r)} \right)^2 \\ = & \; \pi \sum_{m\geq 0} \frac{1}{1+\delta_{m,0}} \left(De_m^{(r)} \right)^2 \\ = & \; \pi \sum_{m\geq 0} \frac{1}{1+\delta_{m,0}} \left(\delta_{r,m}(1+\delta_{r,0}) + \sum_{k\geq 1} \left(A_{m-r}^k + A_{-m-r}^k\right) q^k \right)^2 \\ = & \; \pi \sum_{m\geq 0} \frac{1}{1+\delta_{m,0}} \Bigg(\delta_{r,m}(1+\delta_{r,0})^2 + \sum_{k\geq 1} q^k \Bigg(2\delta_{r,m}(1+\delta_{r,0}) \left(A_{m-r}^k + A_{-m-r}^k\right) \\ & + \sum_{l=1}^{k-1} \left(A_{m-r}^l + A_{-m-r}^l\right) \left(A_{m-r}^{l-k} + A_{-m-r}^{l-k}\right) \Bigg) \Bigg) \end{aligned}$$

$$
\begin{aligned}
&= \pi \sum_{m\geq 0} \left(\delta_{r,m}(1+\delta_{r,0}) + \sum_{k\geq 1} q^k \sum_{l=1}^{k-1} \frac{1}{1+\delta_{m,0}} \left(A^l_{m-r} + A^l_{-m-r}\right)\left(A^{l-k}_{m-r} + A^{l-k}_{-m-r}\right)\right) \\
&= \pi \left((1+\delta_{r,0}) + \sum_{m\geq 0}\sum_{k\geq 1} q^k \sum_{l=1}^{k-1} \frac{1}{1+\delta_{m,0}} \left(A^l_{m-r} + A^l_{-m-r}\right)\left(A^{l-k}_{m-r} + A^{l-k}_{-m-r}\right)\right) \\
&= \pi \left((1+\delta_{r,0}) + \sum_{k\geq 1} q^k \sum_{l=1}^{k-1} \sum_{\substack{m\geq 0 \\ \min(|m-r|,|m+r|)\leq 2\min(l,k-l)}} \frac{1}{1+\delta_{m,0}} \left(A^l_{m-r} + A^l_{-m-r}\right)\left(A^{l-k}_{m-r} + A^{l-k}_{-m-r}\right)\right) \\
&= \pi \left((1+\delta_{r,0}) + \sum_{k\geq 1} q^k \sum_{l=1}^{k-1} \sum_{i=-\min(l,k-l,\lfloor r/2\rfloor)}^{\min(l,k-l)} \frac{1}{1+\delta_{2i+r,0}} \left(A^l_{2i} + A^l_{-2i-2r}\right)\left(A^{l-k}_{2i} + A^{l-k}_{-2i-2r}\right)\right)
\end{aligned}
$$

Note that we have $\min(|m-r|,|m+r|)$ and not $\max(|m-r|,|m+r|)$ in the next to last line. In the last equation, we used the change of variable $2i = m-r$. Equating this with $(1+\delta_{r,0})\pi/C_r^2$ yields the desired result.

p. 3549 equation (45): The correct formula is

$$
\begin{aligned}
\frac{1}{S_r^2} &= 1 + \sum_{k=1}^{\infty}\sum_{l=1}^{k-1} \sum_{i=-\min(k-l,l,\lfloor r/2\rfloor)}^{\min(k-l,l)} \\
&\quad \times \left(B^{k-l}_{2i} - B^{k-l}_{-2i-2r}\right)\left(B^l_{2i} - B^l_{-2i-2r}\right) q^k
\end{aligned}
$$

Proof: Similarly to above, the L_2-norm of the Fourier series in (16b) is

$$
\begin{aligned}
&\int_0^{2\pi} \left(\sum_{m\geq 0} Do^{(r)}_m \sin(mz)\right)^2 dz \\
&= \sum_{m\geq 0} \int_0^{2\pi} \left(Do^{(r)}_m \sin(mz)\right)^2 dz \\
&= \sum_{m\geq 0} \pi \left(Do^{(r)}_m \right)^2 \\
&= \pi \sum_{m\geq 0} \left(\delta_{r,m} + \sum_{k\geq 1} \left(B^k_{m-r} - B^k_{-m-r}\right) q^k \right)^2 \\
&= \pi \sum_{m\geq 0} \left(\delta_{r,m} + \sum_{k\geq 1} q^k \left(2\delta_{r,m} \left(B^k_{m-r} - B^k_{-m-r}\right) \right.\right. \\
&\quad \left.\left. + \sum_{l=1}^{k-1} \left(B^l_{m-r} - B^l_{-m-r}\right)\left(B^{l-k}_{m-r} - B^{l-k}_{-m-r}\right) \right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \pi \sum_{m\geq 0} \left(\delta_{r,m} + \sum_{k\geq 1} q^k \sum_{l=1}^{k-1} \left(B^l_{m-r} - B^l_{-m-r}\right) \left(B^{l-k}_{m-r} - B^{l-k}_{-m-r}\right) \right) \\
&= \pi \left(1 + \sum_{m\geq 0} \sum_{k\geq 1} q^k \sum_{l=1}^{k-1} \left(B^l_{m-r} - B^l_{-m-r}\right) \left(B^{l-k}_{m-r} - B^{l-k}_{-m-r}\right) \right) \\
&= \pi \left(1 + \sum_{k\geq 1} q^k \sum_{l=1}^{k-1} \sum_{\substack{m\geq 0 \\ \min(|m-r|,|m+r|)\leq 2\min(l,k-l)}} \left(B^l_{m-r} - B^l_{-m-r}\right) \left(B^{l-k}_{m-r} - B^{l-k}_{-m-r}\right) \right) \\
&= \pi \left(1 + \sum_{k\geq 1} q^k \sum_{l=1}^{k-1} \sum_{i=-\min(l,k-l,\lfloor r/2\rfloor)}^{\min(l,k-l)} \left(B^l_{2i} - B^l_{-2i-2r}\right) \left(B^{l-k}_{2i} - B^{l-k}_{-2i-2r}\right) \right)
\end{aligned}
$$

and the result follows from equating this with π/S_r^2.

p. 3550 equation (55): For $r = 0$, this formula must be divided by 2 to achieve an L_2-normalization of 2π. Same in equation (58) on the same page.

p. 3550 equation (56): This is actually S_{r+1} and not S_r.