Computer Methods in Applied Mechanics and Engineering 95 (1992) 359–382
North-Holland

Improved accuracy in finite element analysis of Biot's consolidation problem

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Received 26 September 1990

Numerical analysis and error estimates of finite element approximations of Biot's consolidation problem are presented. Initially different orders of interpolation are employed, leading to lower orders of convergence for the pore pressure compared to the displacements of the porous medium. Lower accuracy also occurs in the approximation of the effective stress tensor, whether it is calculated directly from the constitutive equation, or through a primal mixed stress formulation. To improve the rates of convergence of the pore pressure and effective stresses, a sequential Galerkin Petrov–Galerkin post-processing technique is proposed.

1. Introduction

The response of elastic porous media under applied loads consists of an instantaneous deformation followed by a time dependent consolidation process associated with the drainage of the pore fluid. Terzaghi [1] proposed a one-dimensional theory for this problem, deriving a simple model in which the pore pressure satisfies an equation of heat conduction type. Significant progress was made by Biot in several papers [2–5]. In Biot's theory, balance and constitutive laws were proposed for a mixture of a porous linear elastic medium and a Newtonian fluid, whose flow obeys the classical Darcy's law. The resulting evolutionary problem consists of a system of equations, in the pore pressure and the displacement of the solid matrix.

Variational principles for Biot's consolidation problem and finite element approximations based on the Galerkin method are presented in [6–9]. With this formulation, certain combinations of finite element interpolations (including equal order for both fields) are discarded, due to the incompressibility constraint on the displacement field in the initial state. At $t = 0$ we are led to solve an incompressible elasticity or Stokes problem, whose finite element approximation with Lagrangian multipliers has been frequently discussed in the literature [10–14]. It is well known that in classical mixed formulations the finite element spaces must satisfy compatibility conditions in order to fulfill the stability criteria dictated by Babuška–Brezzi theory [15]. For the bidimensional Stokes problem Taylor–Hood elements, with continuous pressure interpolation one order lower than velocity interpolation, are known to be Galerkin stable and quasi-optimally convergent [16–19]. For the consolidation problem, due to the initial condition, Taylor–Hood elements are also employed and seem to be a natural choice. This choice however will obviously lead to an approximation for the pore pressure which is convergent at one order lower than for the displacement field. The same kind of loss of accuracy occurs with the finite element approximation of the effective stress tensor, whether it is calculated through the constitutive equation by differentiating the