Long-Time Behavior of Some Galerkin and Petrov–Galerkin Methods for Thermoelastic Consolidation

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Numerical analysis of some finite element methods for the quasi-static thermoelastic consolidation problem of fluid-filled porous materials is presented for the case of smooth exact solutions. Taking advantage of the exponential decay of the error in the initial data with time, error estimates describing the long-time behavior of the semidiscrete approximation are presented and post-processing techniques are proposed to improve the accuracy of the pore pressure, heat flux, and effective stress approximations. © 1995 John Wiley & Sons, Inc.

I. INTRODUCTION

The classical theory of isothermal quasi-static consolidation of saturated porous media was first formulated by Terzaghi [26] for the one-dimensional case and later extended to the three-dimensional anisotropic continua by Biot [7]. Actually, the hypotheses used in Biot's *ad hoc* structure can be justified from the thermodynamical point of view using the rational approach of the Theory of Mixtures. In this modern context Green and Steel [12] and Crochet and Naghdi [10] by exploitation of the restrictions imposed by the Material Frame Indifference and the Second Law of Thermodynamics, derived constitutive equations for a mixture of an elastic porous medium and a Newtonian fluid, taking into account finite deformations and thermal effects. Biot's theory can be recovered by a linearization of the general problem of the Theory of Mixtures in the neighborhood of a state free of stress.

Thermoelastic consolidation processes are present in many problems in geomechanics such as thermal stresses induced by water-flood technique in secondary oil recovery, underground coal conversion, oil shale retortion, and nuclear waste management. See [23] for illustrative application of the simplified decoupled model. An extended Biot's thermoelastic theory, taking into account the mechanical coupling and neglecting heat convection, is proposed by Schiffman [24]. Kurashighe [15] proposed an extended rational thermoelastic theory for fluid-filled porous materials incorporating heat convection, and applied the model in the simulation of an underground perturbation problem of hot (cold) water injection into an infinite porous medium.

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