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MULTISCALE FLOW AND DEFORMATION IN HYDROPHILIC SWELLING POROUS MEDIA

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Abstract—A three-scale theory of swelling porous media is developed. The colloidal or polymeric sized fraction and vicinal water (water next to the colloids) are considered on the microscale. Hybrid mixture theory is used to upscale the colloids with the vicinal water to form mesoscale swelling particles. The mesoscale particles and bulk phase water (water next to the swelling particles) are then homogenized via an asymptotic expansion technique to form a swelling mixture on the macroscale. The solid phase on the macroscale can be viewed as a porous matrix consisting of swelling porous particles. Two Darcy type laws are developed on the macroscale, each corresponding to a different bulk water connectivity. In one, the bulk water is entrapped by the particles, forming a disconnected system, and in the other the bulk water is connected and flows between particles. In the latter case the homogenized equations give rise to a distributed model with microstructure in which the vicinal water is represented by sources/sinks at the macroscale. The theory is used to construct a three-dimensional model for consolidation of swelling clay soils and new constitutive relations for the stress tensor of the swelling particles are developed. Several heuristic modifications to the classical Terzaghi effective stress principle for granular (non-swelling) media which account for the hydration forces in swelling clay soils recently appeared in the literature. A notable consequence of the theory developed herein is that it provides a rational basis for these modified Terzaghi stresses.

1. INTRODUCTION

Swelling colloids and polymers are ubiquitous in almost all aspects of life. For example, swelling clays act as a source or sink for nutrients and pesticides in agriculture. They play a critical role in various high level nuclear waste isolation scenarios, in barriers for commercial land fills and also in consolidation and failure of foundations, highways and runways. Swelling polymers have numerous technological applications such as drug delivery substrates, in contact lenses, in semiconductor manufacturing and in food stuffs. For simplicity in elucidation of concepts, we henceforth restrict our discussion to swelling clay soils. The reader should be aware however that the theories which we develop herein can easily be applied to most swelling systems, including those involving polymers, other colloids, and food stuffs.

Historically, ad-hoc poroelastic models such as those of Terzaghi [1] and Biot [2] have been developed for consolidation with the aid of experiments conducted at the "macroscale" (Biot and Willis [3]). More recent extensions of the Biot and Terzaghi theories which are more universal in application have been obtained by relaxing some of the restrictive assumptions, such as small deformations and linear elastic constitutive behavior (e.g. finite deformations [4], coupled secondary consolidation and creep [5] and plasticity [6, 7]). As in the Terzaghi and Biot models, the majority of models describing the consolidation of clay have been developed directly at the macroscale, usually by postulating constitutive equations which most accurately reflect the experimental data.

The complex mechanisms underlying the constitutive behavior of a hydrophilic swelling clay soil are a consequence of its complicated microstructure. Due to their tremendous specific surface area and their charged character, clusters of clay platelets when hydrated form "particles" consisting of an assemblage of the platelets (clay minerals) and vicinal (adsorbed) water. These particles swell under hydration and shrink under desication. The platelet–water

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