

A Two-Scale Model for Coupled Electro-Chemo-Mechanical Phenomena and Onsager's Reciprocity Relations in Expansive Clays: I Homogenization Analysis

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Abstract. The macroscopic model governing coupled electro-chemo-mechanical phenomena in expansive clays is revisited within a rigorous homogenization procedure applied to the microscopic governing equations which describe the local interaction between charged clay particles and a binary monovalent aqueous electrolyte solution. The up-scaling of the microscopic electro-hydro-dynamics leads to a two-scale approach wherein the macroscopic model appears governed by a fully coupled form of Onsager's reciprocity relations, mass conservation equations and a modified Terzaghi's effective stress principle. In addition, the two-scale approach provides microscopic representations for the effective coefficients which are exploited herein to obtain further insight in the constitutive behavior of the electrochemical parameters and the swelling pressure. Among other effects, we show that these microscopic closure relations are mainly dictated by the spatial variability of a microscale electric potential which satisfies a local version of the Poisson–Boltzmann problem in a periodic unit cell. The proposed framework allows to address various relevant still open issues regarding the constitutive behavior of swelling systems. Among them we give particular emphasis on the analysis of the influence of the fluctuation and distortion of the electrical double layer upon the magnitude of the electrochemical coefficients and the precise local conditions for the validity of the symmetry of Onsager's relations.

Key words: swelling clay, homogenization, closure problems, Onsager's relations, Poisson–Boltzmann, electro-osmosis, chemico-osmosis, electro-migration, distortion of the double-layer, modified Terzaghi's principle, disjoining (swelling) pressure, streaming current, Hartmann number.

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