

# A Three-Scale Model of pH-Dependent Flows and Ion Transport with Equilibrium Adsorption in Kaolinite Clays: II Effective-Medium Behavior

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**Abstract** In part I (Lima et al., *Transp Porous Media*, 2009), a three-scale model governing the movement of an aqueous saline solution containing four monovalent species ( $\text{Na}^+$ ,  $\text{H}^+$ ,  $\text{Cl}^-$ ,  $\text{OH}^-$ ) in kaolinite clays was derived. Unlike purely macroscopic approaches, the novelty of the formulation relied on the double averaging of the nanoscopic electrochemistry of particle/electrolyte solution interface ruled by the electrical double layer coupled with protonation/deprotonation reactions. The passage from the nano to the micro (pore)-scale gave rise to ion-sorbed concentrations and slip velocity at the solid/fluid interface which are coupled with the microscopic Stokes problem and Nernst–Planck equations governing the hydrodynamics and ion transport in the micropores. Application of a formal homogenization procedure led to macroscopic governing equations with effective electro-chemical parameters, such as retardation coefficients, electro-osmotic permeability, and electric conductivity. In this study, we reconstruct the constitutive laws of the macroscopic coefficients by solving the nano and microscopic closure problems. New generalized isotherms for  $\text{Na}^+$  and  $\text{H}^+ - \text{OH}^-$  sorption are build-up based on a perturbation approach and the limitations

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