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On time averaging in Organised Eddy Simulation modelling

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Abstract: A new technique for devising new models of turbulent viscosity for unsteady quasi-periodic flows is proposed in the report. The turbulent flow simulation strategy is based on two different flowfield decompositions, the Reynolds averaging, and the phase averaging. Properties of the mean flow are recovered on the base of the time-averaging of quasi-periodic solution of the Reynolds averaged Navier-Stokes equations. This yields an upper bound for the turbulent viscosity of the Organised Eddy Simulation model.

The methods under study are verified on a typical test case, the flow around a square cylinder.

Key-words: Turbulence, organised structures, statistical modelling, semi-deterministic models

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Sur la moyenne en temps pour la modélisation en simulation des structures organisées

Résumé : Nous examinons une nouvelle approche pour la modélisation de la viscosité turbulente pour les écoulements quasi-périodiques. Cette approche est fondée sur les deux décompositions classiques, mettant en évidence la moyenne de Reynolds et la moyenne de phase. Certaines propriétés de l'écoulement moyen sont déduites d'un procédé de moyenne en temps. On en déduit un majorant pour la viscosité turbulente des modèles de simulation des structures cohérentes.

La méthode est illustré sur un écoulement typique, celui autour d'un cylindre de section carrée.

Mots-clés : Turbulence, structures organisées, traitement statistique, traitement semi-déterministe

1 Introduction

The very high Reynolds numbers in aircraft flow simulation make impossible the direct calculation of flow parameters around an aircraft. This remark especially applies to the problem of the viscous prediction of flows around aircraft when taking off or landing since both the geometry and the flow are more complex due to high-lift systems. Statistical modelling (SM) , and in practice Reynolds averaging, performs rather well in these cases. It predicts steady flows representing at the same time the statistical and time average models of real turbulent flow. However, there is today a need for the prediction of unsteady structures of the flows under study. Indeed, averaged models are often not always able to predict strongly nonlinear behavior such as separation (or even stall). Also, many flows involve essential unsteadiness that couples with other physics : this is the case of vortex shedding which can excite flutters, or of buffeting which excites some body vibration. Lastly, usual statistical models relying on mean flow modelisation are poorly predictive for many cases for which unsteady approaches may introduce more physics.

Main popular unsteady approaches for rather large Reynolds numbers involve LES (Large Eddy Simulation) and OES (Organized Eddy Simulation). LES approaches aim at providing all or most of the turbulent structures that can be captured by the mesh used, while subgrid structures are damped by an adequate model. Then a LES result is inherently mesh dependant. But the main point is that LES is still today limited to rather low Reynolds flows.

OES is widely described in the literature; we refer to [5], [7], [8], [9], [10], [11], [12], [13] for works related to the application considered in the present work.

OES is a mathematical model and thus should not produce grid dependant results. OES computes an averaged flow, in which averaging is understood in a different manner from statistical modelling, since it is an averaging between instantaneous realisations that corresponds to a same phase with respect to the organised structure. This assumes that the organised structure is quasiperiodic and that the resulting averaged flows is also essentially periodic.

OES has either to be adapted to particular unsteady situations by applying models different from steady ones, or to involve specially designed very complex strategies for accounting both steady and unsteady situations. Indeed, a smooth progression from quasi-periodic to steady can be realised by having an amplitude progressively smaller.

Another important issue for OES models that are based on turbulent viscosity, is to build principles that would allow lower turbulent viscosities than the usual SM ones.

The aim of this paper is to exploit SM principles, in order to derive a new approach allowing the building of OES-type models with lower turbulent viscosities than for SM.

We start from the following remark: depending on flow conditions, a SM "RANS" (Reynolds Averaged Navier-Stokes) simulation (let us say a $k-\varepsilon$ simulation for fix ideas) either produces a steady flow or an unsteady one. If the RANS flow is steady, then the answer is complete since the model is well understood in the case of steady output. If the RANS flow is unsteady, the SM theory cannot be applied. In order to be able to use the unsteady output, it is necessary to invoke the OES theory, assuming the quasi-periodicity of the flow, and relying on phase averaging.

In the proposed work, we suggest to consider the unsteady RANS solution as allowing a good prediction of the time average RANS flow through time averaging:

$$U_{time-averaged} = \frac{1}{T}\int_0^T U_{unsteady-RANS}\, dt$$

This leads to a three-part splitting of the flow:

$$U = U_{time-averaged} + U_{fluctuation-RANS} + U_{other}$$

where

$$U_{fluctuation-RANS} = U_{unsteady-RANS} - U_{time-averaged}$$

and U_{other} contains all fluctuations that are not predicted by the component $U_{fluctuation-RANS}$.

The purpose of this study is to evaluate the possible impact of this splitting on the turbulent viscosity as well as to apply this technique to the actual calculation.

A general computational strategy proposed consists of the following stages.

1. The production of stable quasi-periodic flow component on the base of an initial RANS model.

2. The averaging of gasdynamic parameters (density, velocity, energy) on a period of quasi-periodic solution and, as a result, the production of averaged fields $U_{time-averaged}$.

3. The calculation of steady turbulent viscosity on the base of previously computed averaged fields.

This stage includes the solving of equation system for turbulent energy and dissipation rate. The solution of this linear system with nonlinear source terms is supposed to be stationary.

4. The computation of new gasdynamic fields with the account of stationary turbulent viscosity coefficient and kinetic turbulent energy calculated at the previous stage.

The report is organised as follows:

Section 2 specifies in details the classical RANS model that we consider as the starting point of the present study. Sections 3 and 4 are devoted to some global remarks concerning the relations between RANS and OES. Section 5 proposes the new strategy and Section 6 presents some numerical experiments.

2 The initial RANS model

The classical steady RANS models rely on the following splitting:

$$u_{flow} = \bar{u} + u_{fluctuation}.$$

where $\bar{u}$ is the steady statistical mean of the turbulent flow, that, in practical cases, is also a time-average on a long enough time interval.
As an initial RANS model, the $k - \varepsilon$ model with two-layer formulation of Chen and Patel wall-adjacent turbulence is considered; let us describe it in some details.

2.1 Conservation form of the system

The Reynolds averaged Navier-Stokes system relying to the $k-\varepsilon$ model is written in a conservative form as

$$\frac{\partial W}{\partial t}+\frac{\partial F(W)}{\partial x}+\frac{\partial G(W)}{\partial y}= \frac{1}{\mathrm{Re}}\left(\frac{\partial R(W)}{\partial x}+\frac{\partial S(W)}{\partial y}\right)+\frac{\partial \tilde{R}(W)}{\partial x}+\frac{\partial \tilde{S}(W)}{\partial y}+\Omega(W)$$

where :

- $W(x,y,t)$ is a functional array with values in $I\!R^6$, the components of which are the nondimensionalised conservative variables.
- $F(W)$ and $G(W)$ are the convective flux functions.
- $R(W)$, $S(W)$ are the laminar viscous fluxes. Re is the laminar Reynolds number obtained at the nondimensionalisation.
- $\tilde{R}(W)$, $\tilde{S}(W)$ are turbulent viscous flux functions.
- $\Omega(W)$ is the source term related to the $k-\varepsilon$ model.

Viscous turbulent stress also involves a diagonal term $2/3\rho k\mathcal{I}_d$ ($\mathcal{I}_d$ — the identity matrix) that is accounted through an adhoc variable change:

$$\begin{cases} p^{'}=p+\frac{2}{3}\rho k \\ E^{'}=E+\beta\rho k \quad \text{where} \quad \beta=-1+\frac{2}{3(\gamma-1)} \end{cases}$$

with

$$\begin{cases} p=(\gamma-1)\rho C_v T \\ E=\rho C_v T+\frac{1}{2}\rho(u^2+v^2)+\rho k \end{cases}$$

where p is the pressure, E — the total energy per volume unit, ρ — the density, k — the turbulent kinetic energy, C_v holds of the specific heat for constant volume, T — the temperature, γ — the specific heat ratio assumed as constant

($\gamma = 1.4$ for a perfect gas) and u, v are mean flow velocity components. The relation between $E^{'}$ and $p^{'}$ is classic:

$$p^{'} = (\gamma - 1)\left(E^{'} - \frac{1}{2}\rho\left(u^2 + v^2\right)\right)$$

Then convective fluxes turn to be:

$$F(W) \quad = \quad \begin{pmatrix} \rho u \\ \rho u^2 + p' \\ \rho uv \\ (E' + p')\,u \\ \rho uk \\ \rho u\varepsilon \end{pmatrix} , \quad G(W) \quad = \quad \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p' \\ (E' + p')\,v = \\ \rho vk \\ \rho v\varepsilon \end{pmatrix} .$$

And laminar viscous fluxes are written as: Laminar viscous fluxes are written as

$$R(W) = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ u\,\tau_{xx} + v\,\tau_{xy} + \dfrac{\gamma\,\mu}{\mathrm{Pr}}\dfrac{\partial e}{\partial x} \\ \mu\,\dfrac{\partial k}{\partial x} \\ \mu\,\dfrac{\partial \varepsilon}{\partial x} \end{pmatrix} , \quad S(W) = \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ u\,\tau_{xy} + v\,\tau_{yy} + \dfrac{\gamma\,\mu}{\mathrm{Pr}}\dfrac{\partial e}{\partial y} \\ \mu\,\dfrac{\partial k}{\partial y} \\ \mu\,\dfrac{\partial \varepsilon}{\partial y} \end{pmatrix} .$$

Turbulent viscous fluxes are written as:

$$\tilde{R}(W) = \begin{pmatrix} 0 \\ \tau^t_{xx} \\ \tau^t_{xy} \\ u\,\tau^t_{xx} + v\,\tau^t_{xy} + \dfrac{\gamma\ \mu_t}{Pt}\dfrac{\partial e}{\partial x} + \dfrac{\mu_t}{\sigma_k}\dfrac{\partial k}{\partial x} \\ \dfrac{\mu_t}{\sigma_k}\dfrac{\partial k}{\partial x} \\ \dfrac{\mu_t}{\sigma_\varepsilon}\dfrac{\partial \varepsilon}{\partial x} \end{pmatrix},$$

$$\tilde{S}(W) = \begin{pmatrix} 0 \\ \tau^t_{xy} \\ \tau^t_{yy} \\ u\,\tau^t_{xy} + v\,\tau^t_{yy} + \dfrac{\gamma\ \mu_t}{Pt}\dfrac{\partial e}{\partial y} + \dfrac{\mu_t}{\sigma_k}\dfrac{\partial k}{\partial y} \\ \dfrac{\mu_t}{\sigma_k}\dfrac{\partial k}{\partial y} \\ \dfrac{\mu_t}{\sigma_\varepsilon}\dfrac{\partial \varepsilon}{\partial y} \end{pmatrix},$$

where

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij} \quad , \quad u_1 = u \ , \ u_2 = v$$

$$\tau^t_{ij} = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu_t\frac{\partial u_k}{\partial x_k}\delta_{ij}$$

$$\mathrm{Pr} = 0.725 \quad \mathrm{Pr}_t = 0.86 \quad \mathrm{Re} = \frac{\rho_{ref}\ u_{ref}\ L_{ref}}{\mu_{ref}}$$

The source terms are:

$$\Omega(W) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_k \\ \omega_\varepsilon \end{pmatrix}$$

where

$$\begin{aligned}
\tau_{ij} &= \mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij} \quad , \quad u_1 = u \ , \ u_2 = v \\
\tau_{ij}^t &= \mu_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{2}{3}\mu_t\frac{\partial u_k}{\partial x_k}\delta_{ij} \\
\mathrm{Pr} &= 0.725 \quad \mathrm{Pr}_t = 0.86 \quad \mathrm{Re} = \frac{\rho_{ref}\ u_{ref}\ L_{ref}}{\mu_{ref}}.
\end{aligned}$$

Here μ and μ_t are respectively a nondimensionalised laminar viscosity coefficient, a nondimensionalised turbulent viscosity coefficient, τ_{ij} are the components of the Cauchy stress tensor, Pr and Pr_t are respectively the laminar and turbulent Prandtl numbers obtained from nondimensionalisation. Notations ρ_{ref}, u_{ref}, L_{ref} and μ_{ref} hold respectively for a reference density, a reference velocity a reference length, a reference viscosity. Finally, it is set:

$$\left\{\begin{aligned}
\mu_t &= c_\mu f_\mu \frac{\rho k^2}{\varepsilon} \\
\omega_k &= -\rho\,\varepsilon + \mathcal{P} \\
\omega_\varepsilon &= c_{\varepsilon_1} f_1 \frac{\varepsilon}{k}\mathcal{P} - c_{\varepsilon_2} f_2 \frac{\rho\,\varepsilon^2}{k}
\end{aligned}\right.$$

and the variation of laminar viscosity as a function of dimensional temperature T is defined by the Sutherland law:

$$\begin{cases} \mu(T) &= \mu_{ref} \dfrac{T}{T_{ref}} & \text{if } T \leq 120\ K \\ \mu(T) &= \mu(120) \left(\dfrac{T}{120}\right)^{1.5} \left(\dfrac{120+110}{T+110}\right) & \text{if } T \geq 120\ K \end{cases} \tag{1}$$

The production term is defined by:

$$\mathcal{P} \quad = \quad - \left(\frac{2}{3} \rho k \delta_{ij} - \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right) \right) \frac{\partial u_i}{\partial x_j}$$

where c_μ, c_{ε_1}, c_{ε_2} are constants empirically defined from experiments and f_1, f_2 and f_μ are damping functions.

Constants c_μ, $c_{\varepsilon 1}$, $c_{\varepsilon 2}$, σ_ε, σ_k and the functions f_1, f_2, f_μ are to be specified later.

2.2 Model for field far from wall

The high Reynolds simulation is carried out with the following issues:

$$\begin{cases} f_1 = 1 \\ f_2 = 1 \\ f_\mu = 1 \\ c_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3, \quad c_{\varepsilon_1} = 1.44, \quad c_{\varepsilon_2} = 1.92. \end{cases}$$

2.3 Wall-adjacent model

A two-layer formulation introduced by Chen and Patel in 1988 is chosen. At first, the following numbers are introduced:

$$R_t \quad = \quad \frac{k^2}{\nu_w\ \varepsilon}, \quad R_y \quad = \quad \frac{\sqrt{k}\ y}{\nu_w}, \quad \text{and} \quad y^+ \quad = \quad \frac{u_f\ y}{\nu_w} \tag{2}$$

where ρ_w is the density at wall and ν_w the laminar viscosity at wall.

The one-equation low-Reynolds number model of Wolfshtein is used in the regions near the wall $R_y < 200$

2.3.1 Region of high Reynolds number $R_y > 200$

In this region the standard $k-\varepsilon$ model is used:

$$\frac{D\ \rho k}{D\ t} = \frac{\partial}{\partial x}\left(\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x}\right) + \frac{\partial}{\partial y}\left(\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right) - \rho\ \varepsilon + \mathcal{P}$$

$$\frac{D\ \rho \varepsilon}{D\ t} = \frac{\partial}{\partial x}\left(\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x}\right) + \frac{\partial}{\partial y}\left(\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right) +$$

$$c_{\varepsilon 1}\frac{\varepsilon}{k}\mathcal{P} - c_{\varepsilon_2}\frac{\rho \varepsilon^2}{k}.$$

where $c_\mu = 0.09$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, $c_{\varepsilon_1} = 1.44$, $c_{\varepsilon_2} = 1.92$

and

$$\mu_t = \rho\ c_\mu\ \frac{k^2}{\varepsilon}$$

2.3.2 Region of low Reynolds number $R_y < 200$

In this region only the kinetic energy k equation is solved while dissipation rate ε is derived from a mixing length l_ε:

$$\begin{cases} \dfrac{D\ \rho k}{D\ t} = \dfrac{\partial}{\partial x}\left(\left(\mu + \dfrac{\mu_t}{\sigma_k}\right)\dfrac{\partial k}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(\left(\mu + \dfrac{\mu_t}{\sigma_k}\right)\dfrac{\partial k}{\partial y}\right) - \rho\ \varepsilon + \mathcal{P} \\ \rho\varepsilon = \rho\dfrac{k^{\frac{3}{2}}}{l_\varepsilon} \end{cases}$$

where $\sigma_k = 1.0$ and the turbulent viscosity are given by :

$$\mu_t = c_\mu\ \rho\ \sqrt{k}\ l_\mu, \quad \text{where} \quad c_\mu = 0.09$$

The two mixing length l_μ and l_ε are defined by:

$$l_\mu \quad = \quad \kappa \; {c_\mu}^{-3/4} \; y \; f_\mu \quad \text{and} \quad l_\varepsilon \quad = \quad \kappa \; {c_\mu}^{-3/4} \; y \; f_\varepsilon \tag{3}$$

where f_μ and f_ε are two correction functions:

$$f_\mu \quad = \quad 1 - \exp\left(\frac{-R_y}{A_\mu}\right) \quad \text{and} \quad f_\varepsilon \quad = \quad 1 - \exp\left(\frac{-R_y}{A_\varepsilon}\right) \tag{4}$$

with $\kappa \quad = \quad 0.4$, $A_\mu \quad = \quad 70$ and $A_\varepsilon \quad = \quad 2 \; \kappa \; {c_\mu}^{-3/4}$.

2.3.3 Matching low- and high-Reynolds number regions

For $180 < R_y < 220$ it is assumed that the eddy viscosity varies linearly between the values given by the one-equation and standard $k - \varepsilon$ models:

$$\mu_t \quad = \quad \alpha \mu_t^{k-l} + (1-\alpha)\, \mu_t^{k-\varepsilon}$$

where

$$\alpha \quad = \quad \frac{220 - R_y}{40} \; .$$

3 The OES approach

We concentrate now on the standard context of Organised Eddy Simulation, in which there exists an energy spectrum involving one or several peaks concentrated on particular frequencies, each corresponding to pronounced quasi-periodic behaviors.

Theoretical descriptions of these kind of context can be found in [1],[12],[5].

The OES method consists of splitting the energy spectrum to one discrete part regrouping all the organised modes and coherent motion of the flow system (e.g. the distinct frequency peaks of the spectrum) and to a continuous part, corresponding to the incoherent, chaotic part of the motion (involving for example the continuous part of the spectrum near the peak). For the sake of simplicity, we assume in the sequel that there is only one peak (i.e. one frequency) in the organised part.

The first part of the spectrum is predictable by the unsteady operator of the Navier-Stokes equations, provided a physically suitable averaging with respect to the spectrum decomposition. This is the phase-averaging, which is a measurable quantity and not only a mathematical concept [12]. Let $\hat{u}$ be the phase average of u, the OES formulation relies on the following decomposition:

$$u_{flow} \ = \ \hat{u} \ + \ u_{chaotic}.$$

The continuous part of the spectrum has to be modelled. This part does not regroup only the high wavenumbers as in the case of LES but extends from the low to the high frequency range. Therefore, the criterion of distinction of the structures to be predicted from those to be modelled in OES is their physical nature and not their size (as in the case of LES). Based on these fundamental assumptions, OES is not intrinsically 3D as the LES, but can be 2D when the discrete part of the spectrum corresponds to 2D coherent structures.

In the time-domain, the fundamental assumption of OES leads to employ the decomposition of each unknown quantity in *two parts*, the one being the phase-averaging (which is a time-dependent periodic operator) and the other the random fluctuation.

Therefore the phase-averaged Navier-Stokes equations have the same form as the equations of the statistical averaging (Reynolds averaging), plus the time-dependent term. However, the physical significance of each term is totally different from the statistical averaging equations.

Building new closures in the OES methodology may have as a consequence that the resulting model loose some universality. Indeed, assume that the flow involves a coherent quasi periodic structure that results in a peak of very small amplitude in the energy spectrum, which means that the energy of the periodic structure is very slightly larger than the energy of chaotic ones.

Then both statistical modelling and OES apply. If the OES model is then not identical to the classical RANS one, we can conclude that we meet a limitation of the OES model.

4 Relations between steady RANS and OES

In order to understand the possible impact of the OES approach and its relation to RANS, we consider the decomposition of the instantaneous flow variable u into three components:

$$u_{flow} = \bar{u} + \tilde{u} + u' \ , \tag{5}$$

in which $\bar{u}$ is steady, and $\tilde{u}$ is periodic.

For statistical modelling, the average is the steady flow $\bar{u}$. In phase averaging, the average is the periodic part of the flow:

$$\hat{u} = \bar{u} + \tilde{u}. \tag{6}$$

The averaging processes of both theories lead to essentially to two versions of the same basic model, e.g. $k - \varepsilon$ for their respective averaged flows. In each of these versions, a turbulent energy accounts for the part of the flow that is not in the average flow.

If we consider the energy spectrum of the flow at any point of the flow domain, it appears that the turbulent energy of the steady (RANS) closure is the sum of the analogous one for phase-averaging OES closure with the energy of $\tilde{u}$. As a consequence, the OES energy should be smaller than the RANS one, and the corresponding turbulent viscosities should also satisfy this inequality.

5 A new principle for OES

5.1 Preliminary remarks

Let us assume that flow conditions allow a steady $\bar{u}$, but that the statistical model, after computation, does not produce a steady flow, but a quasi-periodic one. From the phase averaging point of view, it is probable that the model is poorly adapted to the unsteady flow, (or at least to unsteady parts), From the statistical point of view, the unsteady flow has higher gradients than the steady one (who is supposed be derived from averaging), and thus the turbulent viscosity is over-estimated in many parts of the flow domain.

Let suppose that the output flow u_1 is not too bad then $\bar{u}$ can be estimated by *time averaging* which consists in:
- estimating the period,
- computing $\bar{u} = \int u_1 dt$ over the period.

Then new k and ε can be derived from a *steady* closure, and thus a steady ν_{ts} is derived, that would be identical to ν_t in case where amplitude is very small.

We could try to use ν_{ts} for computing a steady $\bar{u}$, but ν_{ts} is (even) smaller than ν_t, and iterative convergence to steady state would be difficult to obtain (this program could be realised by using a continuation process and Newton iteration).

In the unsteady flow $\bar{u} + \tilde{u}$, less turbulent energy has to be modelised; then turbulent viscosity should be even lower.

5.2 Numerical strategy

We shall consider ν_{ts} as a candidate approximation to the OES turbulent viscosity.

The proposed algorithm is as follows:

1. First computation:
   - *compute the flow with a $k - \epsilon$, two-layer model, $C_\mu = 0.09$, obtain an unsteady quasi-periodic flow,*
   - *evaluate period*
   - *derive $\bar{u}$ as the time average.*
2. Reevaluate *steady* closure variables
   - *compute sources terms from $\bar{u}$,*
   - *solve the steady closure system in (k, ε),*
   - *ν_{ts} is also obtained (steady).*
3. Build a new OES model :
   - *compute (time but may be phase) averaged flow,*

- *unknown are* $(\rho, \rho u, \rho v, E)$,
- *closure with* ν_{ts} *(fixed).*

In a first approximation we have chosen to neglect turbulent kinetic energy of the periodic component to total energy.

Step 2 writes as follows:

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho k \bar{u}_i}{\partial x_i} =$$
$$\frac{\partial}{\partial x_i}\left(\bar{\mu}\frac{\partial k}{\partial x_i}\right) + \frac{\partial}{\partial x_i}\left(\frac{\mu_{ts}}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \bar{\mathcal{P}} - \rho\varepsilon$$

$$\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial \rho \varepsilon \bar{u}_i}{\partial x_i} =$$
$$\frac{\partial}{\partial x_i}\left(\bar{\mu}\frac{\partial \varepsilon}{\partial x_i}\right) + \frac{\partial}{\partial x_i}\left(\frac{\mu_{ts}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right) + C_{\varepsilon 1}\frac{\varepsilon}{k}\bar{\mathcal{P}} - C_{\varepsilon_2}\rho\frac{\varepsilon^2}{k}$$

$$\mu_{ts} \; = \; C_\mu \rho \frac{k^2}{\varepsilon}$$

Production term $\bar{\mathcal{P}}$ and laminar viscosity $\bar{\mu}$ are computed from the time average $\bar{u}$.

5.3 Impact on source terms modelling

In accordance to the SM theory for $k - \varepsilon$, the properties of time averaging are used in order to produce turbulent kinetic energy.

The expected effect is to have a lower level of viscosity.

Further, there is to be no more feed back between unsteady structures and turbulent viscosity. Note that this may result in less numerical stabilisation near steep gradients and, thus, finer meshes may be necessary.

5.4 Energetic balance

In accordance to the SM theory for $k-\varepsilon$ models, the turbulent kinetic energy computed from the "steady" mean flow takes into account the whole set of time fluctuations. This means that k involves the energy related both to the organised eddy and to the rest of eddies.

$$k_{turbulent} = k_{RANS-fluctuation} + r_{other}$$

where:

$$k_{RANS-fluctuation} = 0.5\rho_{RANS-fluctuation} \quad ||U_{RANS-fluctuation}||^2$$

Here it is suggested that the splitting corresponds to different parts of an orthogonal spectrum.

The question naturally arising is whether the turbulent viscosity used for computing the RANS-fluctuation should involve the whole kinetic turbulent energy or not.

The option is between either keeping the usual k in the model or subtracting $k_{RANS-fluctuation}$ as evaluated from the RANS fluctuation.

The effect also results in the decreasing of turbulent viscosity. In some cases, this may allow to compute more and more structures in a similar manner as at LES.

6 A numerical experiment

6.1 About numerical method

The numerical technique developed at INRIA is taken as a base for solving the above problem. The solving of compressible Navier-Stokes equations is carried out by the mixed finite volume/element method with the use of a sequence of anisotropic triangular meshes (multigrid strategy).

The Roe scheme of higher accuracy order is used for the approximation of convective terms. P1-formulation of Galerkin method is applied to the discretization of diffusive terms. The time-integrating is performed with the second order of accuracy. To simplify the calculations, the linearized convective fluxes are taken with the first order of accuracy, and to keep the higher accuracy order in the unsteady process, the procedure of DEC correction is used (see [4]).

6.2 Flow past a square cylinder

Our experiments have been concentrated on the 2D flow around a square cylinder at a moderate Reynolds number of 22 000. This test case could be computed without any model, with the DNS approach, but this would be a very expensive 3D computation. It can be also computed with LES ; this approach is also 3D and rather expensive ; in a companion study ([3]), this flow is calculated with a less-expensive version of LES involving wall laws. The present approach aims at presenting OES as a much less expensive option than LES, and it is then interesting to compare them.

We first note that the usual RANS model produce unsteady solutions. In Figure 1, we present the lift coefficient obtained with $k-\varepsilon$ model with $c_\mu = 0.09$. One can see that this result is strong periodic solution which have been one frequency corresponding Strouhal number equal to 0.132 (see,Figure 4).

In Figure 3, there is depicted lift coefficient obtained with new turbulent viscosity where new turbulent viscosity is steady solution of closure equations were computed from time average $\bar{u}$. With the new turbulent viscosity, the flow is much less periodic, showing even some tendency to chaos. The global figures of period and amplitude are not much changed, or even slightly degraded with respect to the RANS output, which were rather good.

The first measures done show that the steady turbulent viscosity deduced from time averaging is smaller than the usual one by a factor k_μ about two. This is shown in Figure 9, in which distribution average and steady turbulent viscosities are plotted near square cylinder.

Let a factor k_μ is

$$k_\mu = \frac{||\mu_t||_{L_2}}{||\mu_{ts}||_{L_2}} \approx 2.43.$$

Here

$$||\mu||_{L_2} = \sqrt{\frac{\sum_{i\in\Omega_i} \mu_i^2 \text{mes}\Omega_i}{\text{mes}\Omega_i}}$$

Then one can define new value of closure coefficient c_μ as

$$c_\mu^{new} = k_\mu c_\mu \approx 0.04$$

The results of computation with new closure coefficient present in Figure 2. One can see that the flow has no clearly defined period and there is a trend to the chaotic motion as it is in the case of steady turbulent viscosity.

Let us pay attention to the appearance of new frequencies in the spectrum of lift coefficient considered as a function on time. These frequencies are conditioned by the generation of additional vortex near the cylinder surface (see Figure 8). In doing so, we obtain an intensity of this additional vortex that is higher in the case of new closure coefficient than the one obtained in applying the classical RANS turbulent viscosity.

As it is seen in the Figures 4-5 the energy of the main vortex (corresponding to the quasiperiodic flow) decreases and the process of energy redistribution for new structures generation starts. In the case of new value of closure coefficient this process is taking the higher intensity.

It seems interesting to compare the results with the ones obtained by a LES calculation ([3]). The corresponding spectrum correlation (Figures 6-7) shows a good agreement both in the additional frequencies and their energies in the case of using $k - \varepsilon$ turbulence model with new value c_μ coefficient.

The time for obtaining a significative output is rather small, about ten periods, which indicates that the proposed method needs a moderate computing time. The ratio with a LES calculation with comparable accuracy is nearly two orders of magnitude.

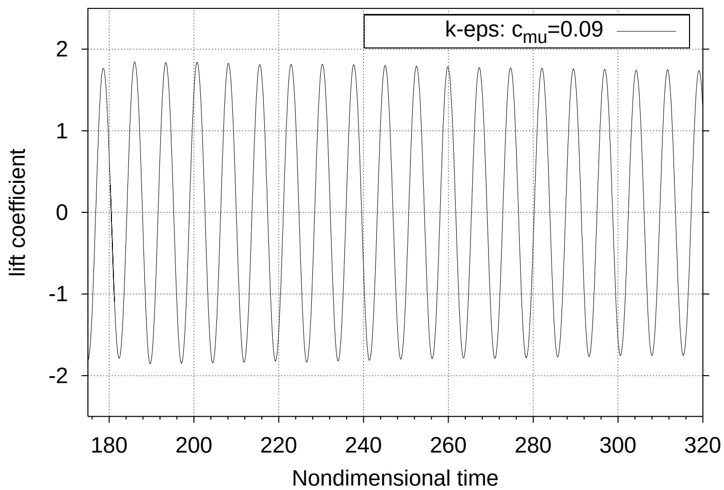

Figure 1: Lift coefficient as function of nondimensional time obtained with $k-\varepsilon$ equations model with $c_\mu = 0.09$

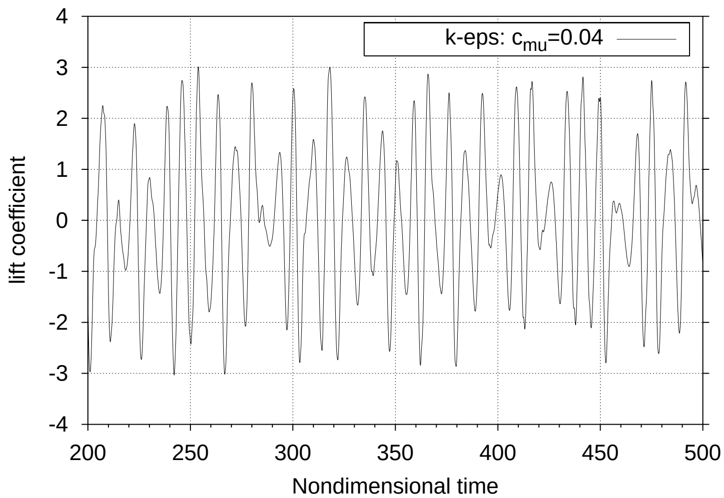

Figure 2: Lift coefficient as function of nondimensional time obtained with $k-\varepsilon$ equations model with $c_\mu = 0.04$

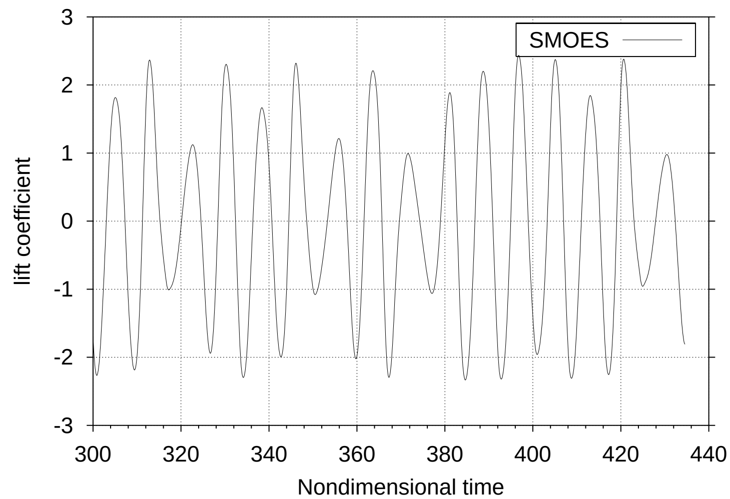

Figure 3: Lift coefficient as function of nondimensional time obtained with SMOES

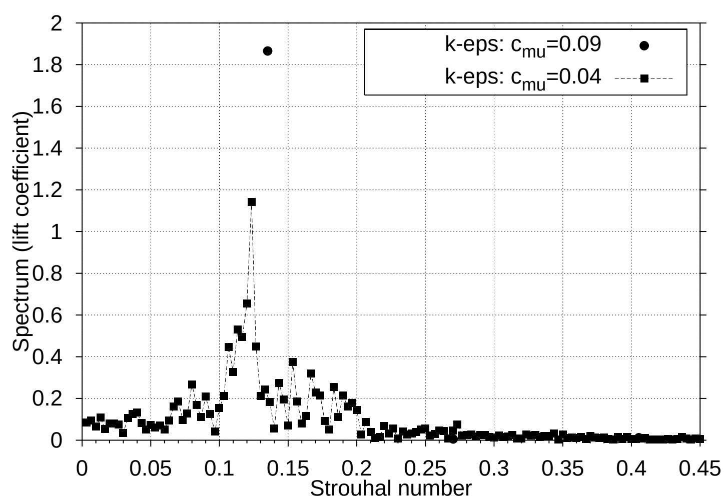

Figure 4: Comparison lift coefficient spectrums obtained with $k-\varepsilon$ equations model with two different values of c_μ: 0.04 and 0.09

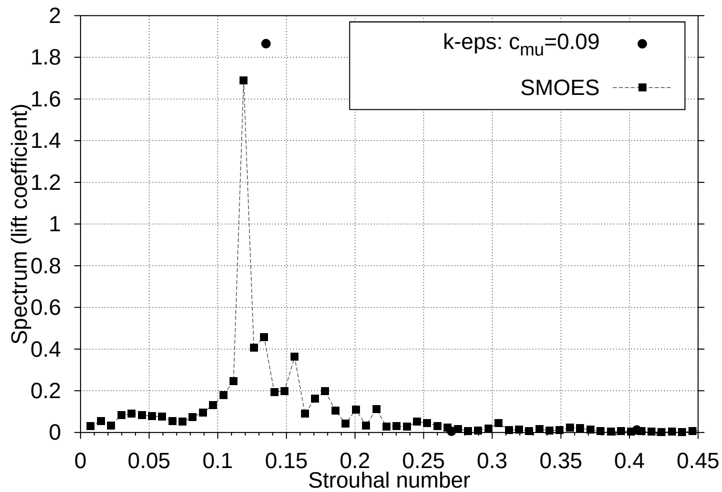

Figure 5: Comparison lift coefficient spectrums obtained with SMOES and $k - \varepsilon$ equation model ($c_\mu = 0.09$)

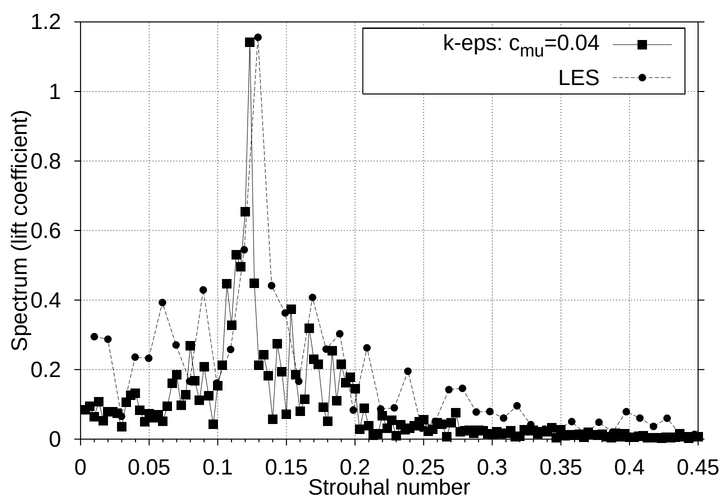

Figure 6: Comparison lift coefficient spectrums obtained with LES and $k - \varepsilon$ equation model ($c_\mu = 0.04$)

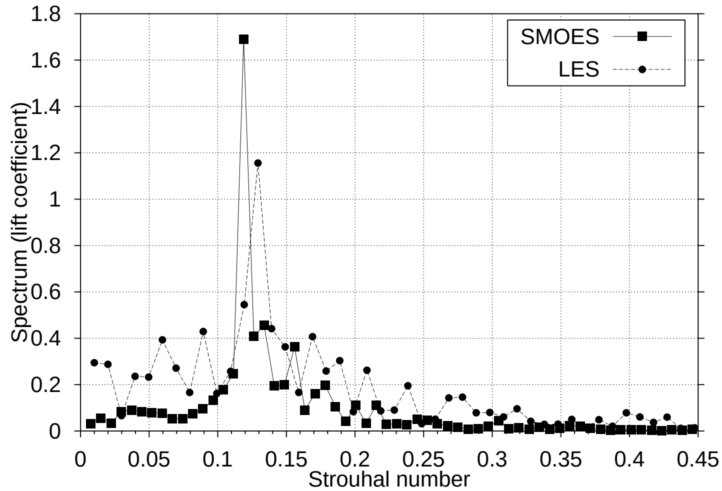

Figure 7: Comparison lift coefficient spectrums obtained with SMOES and LES

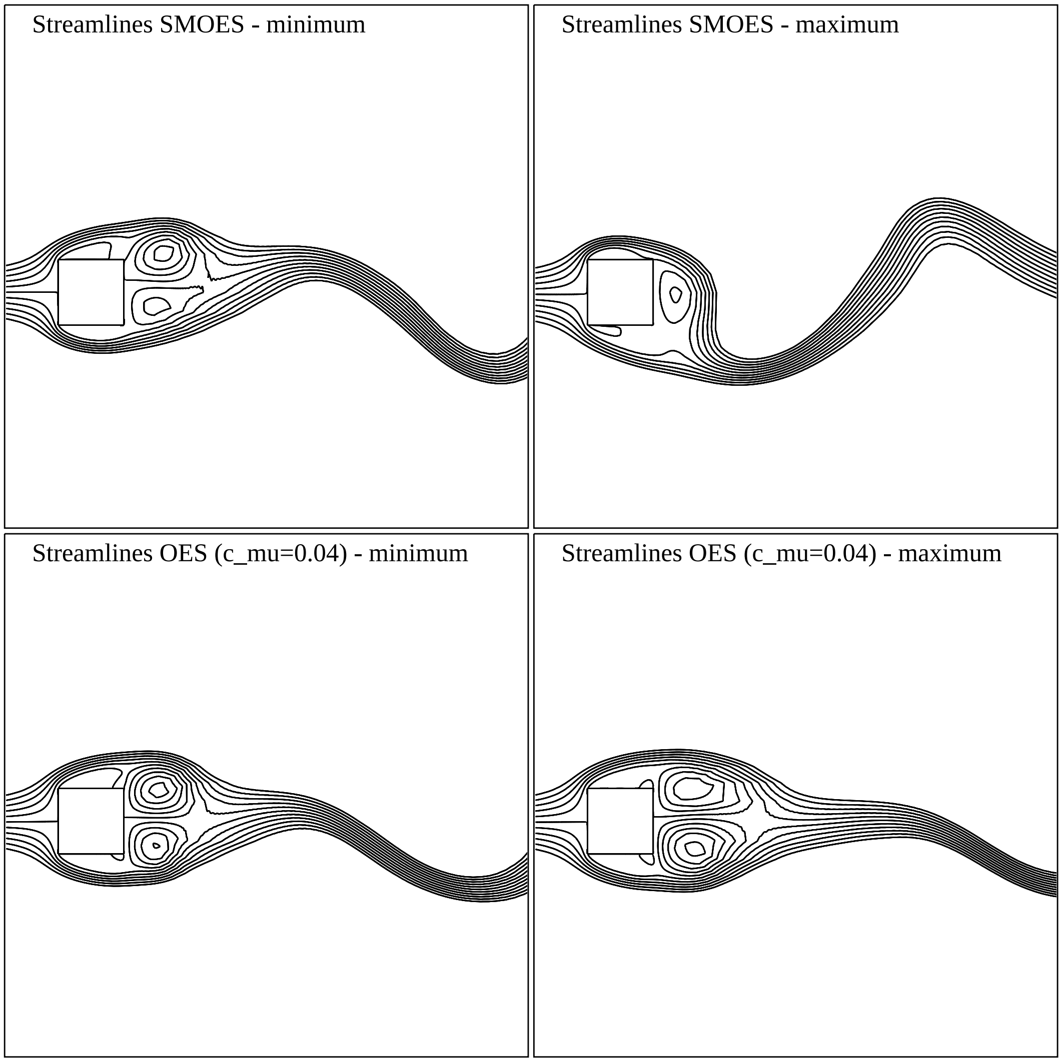

Figure 8: Streamlines at the time corresponding minimal and maximal values of lift coefficient; the upper part is calculations on $k-\varepsilon$ model with $c_\mu = 0.04$; the lower part is calculations on SMOES

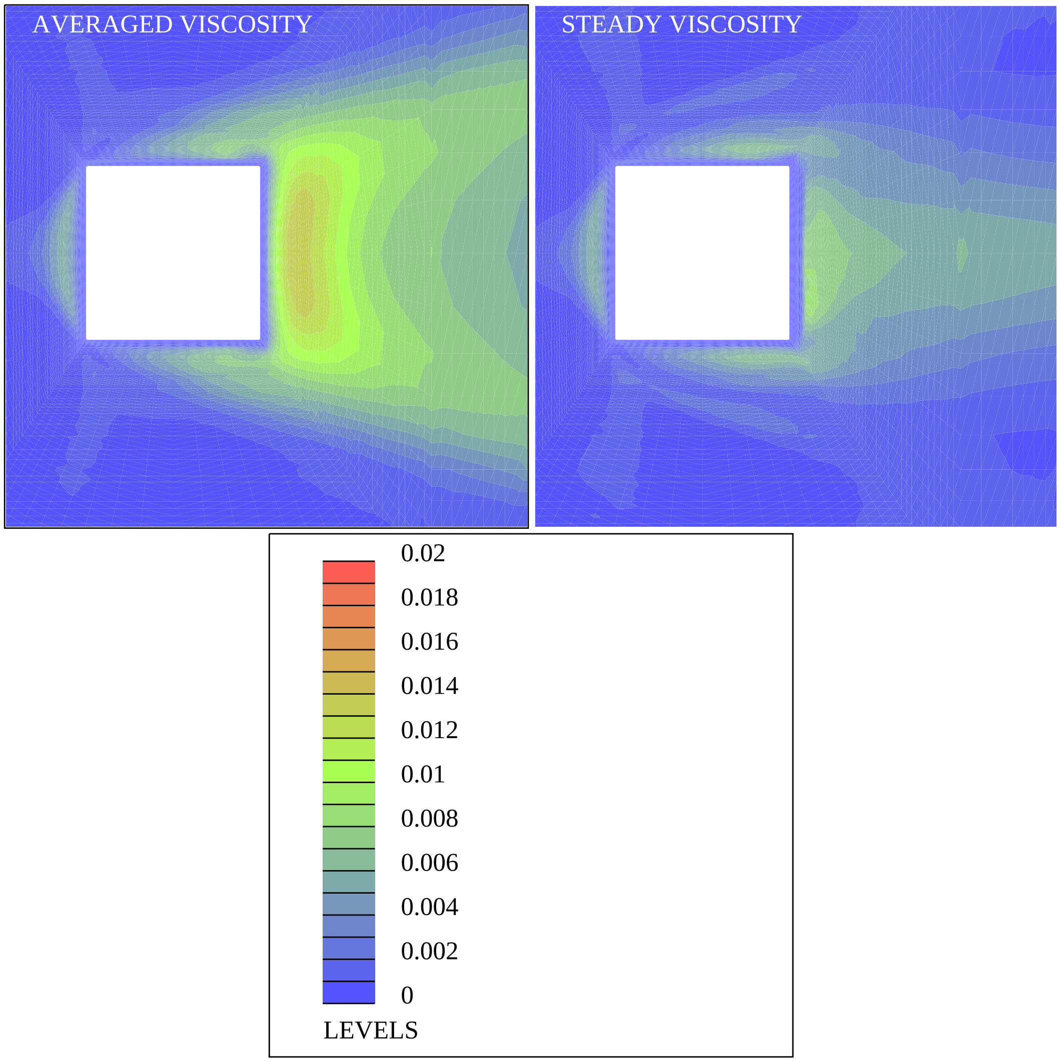

Figure 9: Comparison eddy viscosity obtained with averaged by period of quasiperiodic solution (ν_t) and obtained with solution of the $k-\varepsilon$ equations with mean velocities (steady eddy viscosity ν_{ts})

7 Conclusion

This short study has concentrated on the use of *steady* turbulent viscosity for OES modelling.
One novelty is that this model is by construction consistent with the calculation of steady mean flows; indeed, the level of viscosity is different from SM models as soon as unsteadiness appears and is the same when unsteadiness does not appear.
However, when SM provides a good prediction, the new model provides less periodic outputs, with an accuracy of prediction a little degraded.

In the proposed square cylinder example, we start with a RANS calculation that is good enough for predicting rather well the quasi periodic flow. In many other cases, the classical RANS calculation cannot provide such an unsteady flow. It is then necessary to consider a first trial relying on a modified RANS model, in which the turbulent viscosity has been lowered by an arbitrary factor. It is clear that the resulting level of viscosity should take into account the lowest level permitted by the mesh, i.e. the viscosity should be large enough for filtering structures smaller that the mesh size. Another approach could be of course to compute first the flow with a LES model, derive then a mean flow and obtain with our approach an OES formulation. The question is whether we can extract general features of it in order to be able to apply it to other flows than the initial one, for exemple flows with a different Reynolds number.

Many extra computations are still necessary to decide whether the proposed approach can be useful for the simulation of quasi-steady/quasi-periodic flows as arising in many engineering problems. The sequel of our work will concern quasi-periodic flows around airfoils at high angle of attack.

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