FREQUENCY WARPED BURG'S METHOD FOR AR-MODELING

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ABSTRACT

Auto-regressive modeling of measured data is commonly used in numerous signal processing applications. When aiming for high accuracy Burg's method has been found to give a suitable model. It has been shown that when the signal energy is non-uniformly distributed in frequency range, the use of a modified frequency scale is advantageous. This is often the case with audio signals. In this paper we introduce a frequency-warped version of Burg's method for calculating the auto-regressive filter parameters. A bilinear frequency mapping can be embedded in Burg's method by replacing the unit-delays of the lattice structure used in Burg's method with first-order allpass filters. The benefits of the frequency-warped Burg's method are demonstrated by comparing its signal modeling performance against those of the conventional Burg's method and the warped Yule-Walker method.

1. INTRODUCTION

An auto-regressive (AR) model [1] is defined by equation

$$y_n = -\sum_{m=1}^{p} a_m y_{n-m} + e_n \tag{1}$$

where y_n are the signal samples, p is the model order, a_m are the model coefficients, and e_n is the residual. The model coefficients a_m are calculated by minimizing the total energy of the residual $E = \sum_n e_n^2$. There exist several methods for estimating the AR parameters. The least squares method (also known as the covariance method) and the Yule-Walker method (also known as the autocorrelation method) are the mostly used approaches for historical reasons [2]. Burg's method is considered preferable for applications which require models of high accuracy, e.g., signal extrapolation [3] and detection [2].

There is a long tradition in performing signal analysis and processing on a warped frequency scale. For example, a non-uniform resolution Fourier transform technique that uses first-order allpass filters has been presented in [4]. Moreover, signal modeling and linear prediction on a warped scale have been proposed in [5] as a way of approximating the frequency resolution of the human auditory system. See [6] for a comprehensive historical background.

Significant benefit is gained from frequency warping in AR modeling when the energy distribution of the signal is concentrated on the lower or higher frequency range. Previously, a frequency-warped version of the Yule-Walker method has been employed successfully in several audio-related applications [6]. Other applications of frequency warping include analysis, synthesis, and de-noising of audio signals [7].

In this paper we show that frequency-warping can be combined with Burg's method in a straightforward way and introduce the warped Burg's algorithm. We apply the warped Burg's algorithm to signal modeling and show that while using the same number of model coefficients significant enhancements are obtained in modeling accuracy in a perceptual sense.

The rest of the paper is organized as follows. In Section 2, we present the conventional Burg's algorithm. In Section 3, the principle of frequency warping is discussed. In Section 4, the warped Burg's algorithm is derived and in Section 5, it is applied to signal modeling. Conclusions are drawn in Section 6.

2. BURG'S ALGORITHM

From Eq. (1) it is easily seen that the residual e_n can be calculated from the signal y_n by

$$e_n = y_n + \sum_{m=1}^{p} a_m y_{n-m} = \sum_{m=0}^{p} a_m y_{n-m} \tag{2}$$

where $a_0 = 1$. If the signal frame consists of N samples y_0, $y_1, \ldots, y_{N-1}$, the residual samples $e_p, e_{p+1}, \ldots, e_{N-1}$ can be regarded as the output of a finite impulse response (FIR) prediction error filter. This FIR filter can be implemented through the lattice structure shown in Fig. 1. The equations of the lattice filter are

$$\begin{aligned} f_n^{(l)} &= f_n^{(l-1)} + k_l b_{n-1}^{(l-1)} \\ b_n^{(l)} &= b_{n-1}^{(l-1)} + k_l f_n^{(l-1)} \end{aligned} \qquad n = l, l+1, \ldots, N-1 \tag{3}$$

where $f_n^{(l)}$ and $b_n^{(l)}$ are the forward and backward prediction errors and k_l are the reflection coefficients of the stage l. The initial values for the residuals are $f_n^{(0)} = b_n^{(0)} = y_n$. Burg's algorithm calculates the reflection coefficients k_l so that they minimize the sum of the forward and backward residual errors [8]. This implies an assumption that the same AR coefficients can predict the signal forward and backward. The sum of residual energies in stage l is

$$E_l = \sum_{n=l}^{N-1} \left(f_n^{(l)}\right)^2 + \left(b_n^{(l)}\right)^2 . \tag{4}$$

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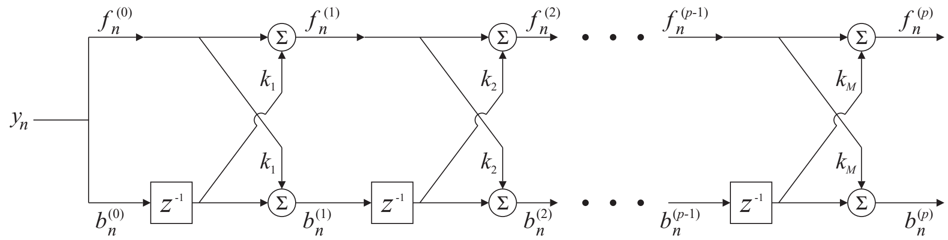

Figure 1: Prediction-error filter with a lattice filter structure.

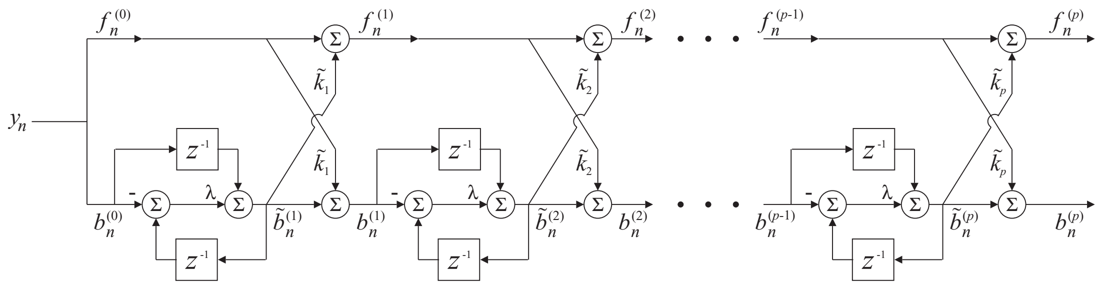

Figure 2: Lattice form of the warped prediction-error filter. The unit delays in Fig. 1 have been replaced by first-order allpass filters.

Minimizing E_l with respect to the reflection coefficient k_l yields

$$\frac{\partial E_l}{\partial k_l} = 2\sum_{n=l}^{N-1}\left\{\left(f_n^{(l-1)} + k_l b_{n-1}^{(l-1)}\right) b_{n-1}^{(l-1)} + \left(b_{n-1}^{(l-1)} + k_l f_n^{(l-1)}\right) f_n^{(l-1)}\right\} = 0, \tag{5}$$

from which the reflection coefficients can be solved, i.e.,

$$k_l = \frac{-2\sum_{n=l}^{N-1} f_n^{(l-1)} b_{n-1}^{(l-1)}}{\sum_{n=l}^{N-1}\left(f_n^{(l-1)}\right)^2 + \left(b_{n-1}^{(l-1)}\right)^2}. \tag{6}$$

The AR coefficients a_m can be obtained from the reflection coefficients k_l via the Levinson-Durbin algorithm. The recursion is initialized with $a_0^{(0)} = 1$ and

$$a_m^{(l)} = a_m^{(l-1)} + k_l a_{l-m}^{(l-1)} \qquad m = 1, 2, \ldots, l-1 \tag{7}$$

$$a_l^{(l)} = k_l \tag{8}$$

is repeated for $l = 1, 2, \ldots, p$. At the end of the iterations, $a_m^{(p)}$ gives the desired prediction error filter coefficients a_m of Eq. (2). Equation (6) ensures that $|k_l| < 1$ and therefore Burg's method is guaranteed to provide a stable model.

3. FREQUENCY WARPING

The time-domain representation of a signal relates to its spectrum via the Fourier transform. The frequency-resolution of the resulting spectrum is uniform along the frequency axis. Signal analysis on non-uniform frequency-resolutions or on frequency-warped scales can be achieved by means of a frequency mapping operator.

In this paper, frequency warping is restricted to a conformal bilinear mapping. This basically means that the unit-delays, z^{-1}, of the employed filter structures are replaced with first-order allpass filters, $D(z)$. These allpass filters can be regarded as frequency-dependent delay elements and are defined by

$$\tilde{z}^{-1} = D(z) = \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}}. \tag{9}$$

Conversely to the linear phase response of an ordinary unit-delay, the phase response of $D(z)$ can be made non-linear by adjusting the warping factor parameter λ. Indeed, the mapping from the uniform to the warped frequency scale is governed by the phase response of $D(z)$, which is given by [5]

$$\tilde{\omega} = \arctan\left\{\frac{(1-\lambda^2)\sin(\omega)}{(1+\lambda^2)\cos(\omega) - 2\lambda}\right\}, \tag{10}$$

where $\omega = 2\pi f/f_{\mathrm{s}}$ and f_{s} is the sampling frequency. Figure 3 shows the attained mapping for several values of λ. For positive values of λ, the resolution at low frequencies is increased. On the contrary, negative values of λ yield a higher resolution at high frequencies. Suitable values of λ can be chosen depending on the application. For instance, in [9] it is shown that an approximation of the frequency resolution of the human auditory system is attained by setting $\lambda = 0.723$.

Warped linear predictive coding can be carried out similarly to standard methods. For instance, the coefficients $\tilde{a}_m$ of a warped prediction filter can be estimated via the warped autocorrelation normal equations. In these equations, the conventional autocorrelation function $r_k = \mathrm{E}\{y_n y_{n-k}^*\}$ is replaced with

$$\tilde{r}_k = \mathrm{E}\{\tilde{\delta}_0[y_n]\ \tilde{\delta}_k[y_n^*]\}, \tag{11}$$

where E is the expectation operator and $\tilde{\delta}_k[\cdot]$ is a generalized shift operator defined by [6]

$$\tilde{\delta}_k[y_n] = \underbrace{d_n * d_n * \cdots * d_n}_{k \text{ fold convolutions}} * y_n, \tag{12}$$

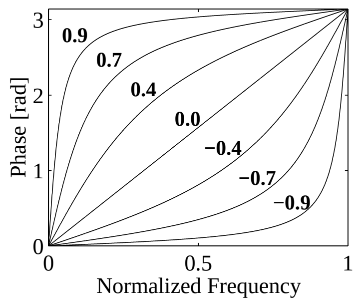

Figure 3: Phase response of $D(z)$ for several values of λ.

with d_n being the impulse response of the allpass filter. Yet, the equation system can be solved efficiently via the Levinson-Durbin algorithm. Finally, the prediction error filter is given by $A(z) = \sum_{m=1}^{p} \tilde{a}_m D(z)^m$. See [6] for more detailed information.

4. WARPED BURG'S ALGORITHM

The warped Burg's method is based on warping the lattice filter depicted in Fig. 1. This is done by replacing the delay elements with warping allpass filters. The warped prediction error filter is shown in Fig. 2. To calculate the warped prediction error in stage l we need the allpass filtered backward residual

$$\begin{aligned} \tilde{b}_n^{(l)} &= b_{n-1}^{(l-1)} - \lambda \left[b_n^{(l-1)} - \tilde{b}_{n-1}^{(l)} \right] \\ n &= l, l+1, \ldots, N-1, \end{aligned} \tag{13}$$

where λ is the warping factor. Because this is a recursive filter the initial condition (i.e. the value of $\tilde{b}_{l-1}^{(l)}$) has to be set. Using $\tilde{b}_{l-1}^{(l)} = 0$ is the most obvious choice.

Warping also changes the lattice equations of Eq. (3) to

$$\begin{aligned} f_n^{(l)} &= f_n^{(l-1)} + \tilde{k}_l \tilde{b}_n^{(l)} \\ b_n^{(l)} &= \tilde{b}_n^{(l)} \quad + \tilde{k}_l f_n^{(l-1)}. \end{aligned} \qquad n = l, l+1, \ldots, N-1 \tag{14}$$

The resulting equation for the reflection coefficient is

$$\tilde{k}_l = \frac{-2 \sum_{n=l}^{N-1} f_n^{(l-1)} \tilde{b}_n^{(l)}}{\sum_{n=l}^{N-1} \left(f_n^{(l-1)} \right)^2 + \left(\tilde{b}_n^{(l)} \right)^2}. \tag{15}$$

From Eq. (13) it can be seen that parameter value $\lambda = 0$ reduces the algorithm to ordinary Burg's method.

5. EXPERIMENTS

Modeling signals with all-pole infinite impulse resonse (IIR) filter is a powerful application of AR modeling. One example of this is linear predictive coding (LPC) which is usually used to model only the spectral envelope of the signal. In this section the modeling performance of different methods is compared with emphasis on high perceptual accuracy. The target is to obtain a model that extracts the strongest signal frequencies. This is an important issue, e.g., in model-based signal extrapolation.

Figures 4–7 present a comparison among the ordinary Burg's method, its warped version, and the warped Yule-Walker method.

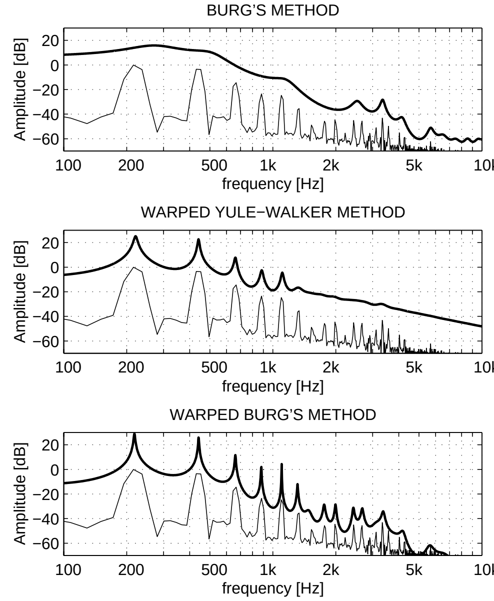

Figure 4: Comparison among the magnitude responses of the all-pole filters obtained using Burg's method, the warped Yule-Walker method, and the warped Burg's method. The employed setup was $p = 50$ and $\lambda = 0.723$. The thinner curves relate to the signal whereas the thicker (shifted upwards for clarity) to the models.

The comparison is made using the same model order $p = 50$ in all three methods and warping factor $\lambda = 0.723$ in both of the warped methods to provide frequency-warping close to the auditory frequency scale [9]. Figure 4 shows the magnitude spectrum of a target signal (2048 samples of a guitar tone sampled at $44.1\,\mathrm{kHz}$) and those of the modeled signals, which were obtained using different methods. Figures 5 and 6 compare the same models in the z-plane by showing the pole locations of the models. In the frequency domain, the ordinary Burg's method models the signal with equal emphasis on all the frequencies. In the example, the target signal has most of its energy concentrated in the lower quarter of the (linear) frequency range but the actual model has its poles equally concentrated across the whole frequency range. Burg's method

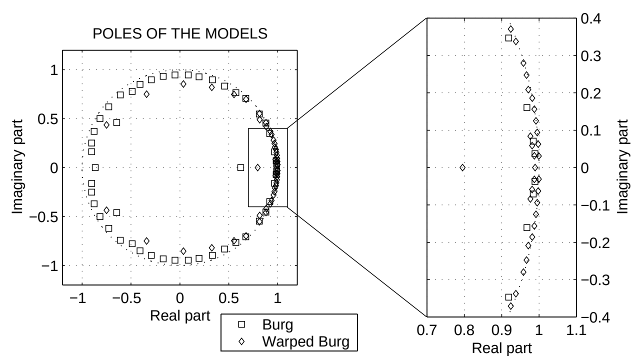

Figure 5: The poles of the AR-models obtained using Burg's method and the warped Burg's method. The employed setup was $p = 50$ and $\lambda = 0.723$.

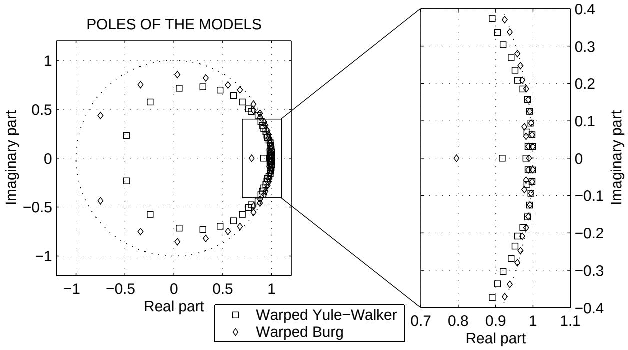

Figure 6: The poles of the AR-models obtained using the warped Burg's method and the warped Yule-Walker method. The employed setup was $p = 50$ and $\lambda = 0.723$.

"wastes" its poles by modeling the high frequency noise peaks. By using positive warping factor λ, the emphasis of the modeling procedure is shifted to lower frequencies, thus resulting in a more accurate model in a perceptual sense with the same model order. If the same accuracy in lower frequency region were to be achieved with the ordinary Burg's method the model order should be about three times higher. For equal comparison in terms of computational cost the order should be only about 33 % higher to compensate for the more expensive modeling. The warped Yule-Walker method captures the most significant spectral components of the signal but the model obtained is still less accurate than the one given by the warped Burg's method. The warped Burg's method places the poles closer to the unit circle and this makes the peaks more pronounced in the frequency response of the all-pole filters.

Figure 7 compares the spectra of the residuals obtained with different methods. In the lower frequencies, the ordinary Burg's method models only the spectral envelope of the signal. As a result the ratio of the height of the spectral peaks to the background is preserved in the residual. The warped Yule-Walker method models also some of the strongest spectral peaks and in the residual the peaks rise lower above the background. The residual obtained with the warped Burg's method has a more flat spectrum in a logarithmic scale than the ones obtained with the other methods.

6. CONCLUSIONS

In this paper we have presented the warped Burg's algorithm for calculating the AR model parameters. The warping is achieved by replacing the delay elements of the lattice filter by first-order allpass warping elements.

We have demonstrated the improving effect of warping in modeling of audio signals whose energy distribution is concentrated in the lower part of the spectral range. The improved AR modeling of the warped Burg's method is compared to the conventional Burg's method and the warped Yule-Walker method. It can be concluded that, for a given model order, the warped Burg's method yields more accurate models from the perceptual point of view.

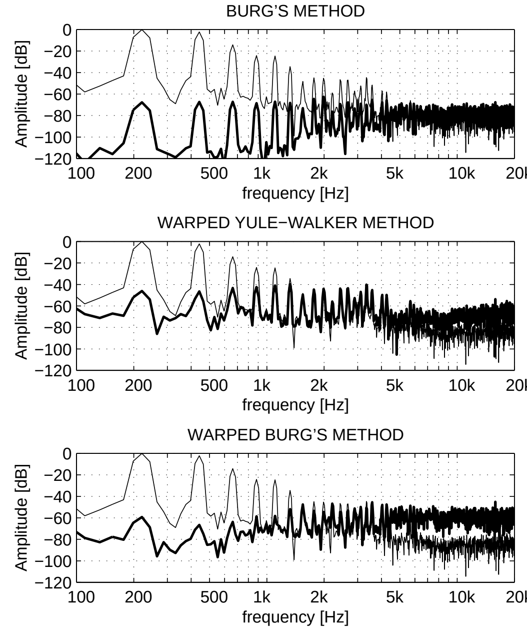

Figure 7: Comparison among the magnitude spectra (thicker curves) of the prediction residuals obtained using Burg's method, the warped Yule-Walker method, and the warped Burg's method. The employed setup was $p = 50$ and $\lambda = 0.723$. The thinner curves represent the magnitude spectrum of the signal.

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