A GENETIC ALGORITHM FOR MULTIOBJECTIVE STRUCTURAL OPTIMIZATION

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ABSTRACT

A genetic algorithm for multiobjective optimization is presented which tries to evolve an evenly distributed set of solutions belonging to the Pareto set by: (i) ranking the population according to nondomination properties; (ii) defining a filter to retain Pareto set solutions and (iii) using adequate operators: exclusion, addition and single-objective operator which improves the individuals from the current filter in order to achieve a better distribution of solutions along the Pareto set.
Numerical experiments are presented in order to illustrate the performance of the proposed algorithm when applied to multiobjective optimization problems in structural mechanics.

1. INTRODUCTION

Virtually all design problems are actually multiobjective optimization problems (MOOP) that due to the lack of a better methodology and adequate computational procedures are simplified by aggregating - usually via a convex combination - all the objectives into a single one. Those objectives are usually conflicting and often incommensurable and there is not a single optimum solution to the problem when all objectives are considered. The set of solutions which are such that no improvement can be made in one objective without deteriorating at least one of the other objectives is called the Pareto set of non-dominated solutions and an approximation of it would be very useful in order to get insight into the problem and assist the decision making process.

Genetic algorithms (GAs) are search procedures inspired by natural selection which have features that make them attractive for the construction of the Pareto set in multiobjective optimization problems: they are population-based, require only objective function values, and use probabilistic transition rules which make them less prone to local optimum entrapment. Besides those features, GAs are naturally parallel allowing for the implementation in multi-processor architectures as well as in clusters of workstations and/or personal computers.

2. PARETO MULTIOBJECTIVE GENETIC ALGORITHM

The Pareto Multiobjective Genetic Algorithm (PMOGA) proposed here makes use of a filter to retain Pareto set solutions as presented in [Cheng and Li, 1998]. However, in order to improve the results of the MOOP, three new operators – the exclusion, the addition and the single-objective operator – are introduced here.

A strategy of ranking based in nondomination properties is used. The non-dominated solutions receive the rank 1, then a new nondomination test is made in the population without these solutions and the new non-dominated solutions receive the rank 2. This procedure is repeated until all solutions are ranked. After that, a tournament selection based in ranking position is used and the evolution of the population continues.

Due to the existence of more than one objective, the search space is more complex and a higher mutation rate can be used in order to test more dispersed solutions, thus better exploring the search space.

The exclusion operator introduced here finds the closest solution in the current filter to be removed in order to achieve a better distribution of solutions along the Pareto set. This exclusion procedure is repeated until the current filter reaches the specified size. The metric used can be written as:

$$d_{j,k} = \sum_{i=1}^{N.fo} \frac{100.\left| fo_i(j) - fo_i(k) \right|}{0,5.[\, fo_i(j) + fo_i(k)]} \tag{1}$$

This metric results in a process of sum of the percentages of the distances of each objective between the two solutions *j* and *k*.

The addition operator finds the two more distant (in the objective function space) solutions and recombines them *n* times, where the value of *n* is usually between 2 and 6. The idea of this operator is to find solutions which fill the space between the two far away solutions.

Finally, the single-objective operator finds the two extreme solutions, according to each objective, in order to perform *n* recombinations between them. The idea is to better search for the optimum of each individual objective thus obtaining a better distribution of the solutions along the Pareto set.

These three operators work at the end of each generation. However, it is possible and in fact recommended that, after a certain number of generations are processed and a significant number of Pareto solutions are retained in the filter, additional generations should be processed using only these three operators. These additional generations are faster than the standard ones and they significantly improve the quality of the final Pareto set.

3. APPLICATIONS

To illustrate the use of the proposed PMOGA, two engineering design examples from the literature were selected. Although a real-coded version of the PMOGA can also be conceived following the same ideas, the standard binary coding was used here. The recombination operator adopted was the uniform crossover applied with a probability of 0.85. The mutation rate was set at 0.05 and the selection procedure used was a tournament based on the nondomination ranking. In both examples the PMOGA was run only once, starting from a randomly generated initial population.

3.1. Example 1: Design of an I-beam

In this problem it is necessary to find the dimensions of the beam sketched in Figure 1, that satisfy geometric and strength constraints and minimize the following conflicting objectives:

- cross section area of the beam;
- static deflection of the beam under the vertical load P.

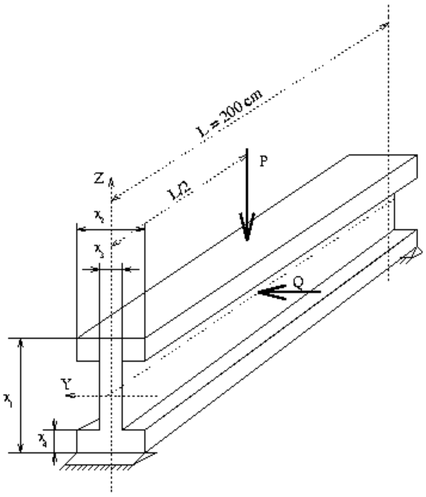

Figure 1: The simply supported I-beam [Coello and Christiansen, 1997].

It is assumed that:

- $E = 2 \times 10^4$ kN/cm^2 (Young's Modulus of Elasticity);
- $k_g = 16$ kN/cm^2 (permissible bending stress of the beam material);
- P = 600 kN and Q = 50 kN (maximal bending forces).

The vector of decision variables is $\mathbf{x} = [\ x_1,\ x_2,\ x_3,\ x_4\ ]^T$. Their values will be given in centimeters and the geometric constraints are:

$$10 \le x_1 \le 80; \quad 10 \le x_2 \le 50; \quad 0{,}9 \le x_3 \le 5; \quad 0{,}9 \le x_4 \le 5 \tag{2}$$

The strength constraint is

$$\frac{M_y}{W_y} + \frac{M_z}{W_z} \le k_g \tag{3}$$

where:

- M_y and M_z are maximal bending moments in the Y and Z directions;
- W_y and W_z are section module in the Y and Z directions.

The maximal bending moments due to the loads P and Q are M_y=30000kN.cm and M_z=2500kN.cm respectively; and the section modules can be expressed as follows:

$$W_y = \frac{x_3(x_1 - 2x_4)^3 + 2x_2x_4\left[4x_4^{\ 2} + 3x_1(x_1 - 2x_4)\right]}{6x_1} \tag{4}$$

$$W_z = \frac{(x_1 - 2x_4)x_3^{\ 3} + 2x_4x_2^{\ 3}}{6x_2} \tag{5}$$

Thus the strength constraint is:

$$16-\frac{180000x_1}{x_3(x_1-2x_4)^3+2x_2x_4\left[4x_4^{\ 2}+3x_1(x_1-2x_4)\right]}-\frac{15000x_2}{(x_1-2x_4)x_3^{\ 3}+2x_4x_2^{\ 3}}\geq 0 \qquad (6)$$

Finally, the objective functions are:

- Cross section area

$$f_1\,(\mathbf{x}) = 2x_2x_4+x_3(x_1-2x_4)\ \text{cm}^2 \qquad (7)$$

- Static deflection

$$f_2(x)=\frac{PL^3}{48EI}cm \qquad (8)$$

where I is the moment of inertia which can be calculated from

$$I=\frac{x_3(x_1-2x_4)^3+2x_2x_4\left[4x_4^{\ 2}+3x_1(x_1-2x_4)\right]}{12} \qquad (9)$$

After substitutions, the second objective function is:

$$f_2(x)=\frac{60000}{x_3(x_1-2x_4)^3+2x_2x_4\left[4x_4^{\ 2}+3x_1(x_1-2x_4)\right]}cm \qquad (10)$$

For this example the length of the chromosome is 43 and a simple penalty method is applied: for the infeasible points the constants c_1 = 1000 e c_2 = 10 are added to the objectives 1 and 2 respectively, as follow:

$$f_i\,(\text{x}) = f_i\,(\text{x}) + c_i \qquad (11)$$

A population of 50 individuals is allowed to evolve for 50 generations. The number of additional generations that act only on the filter is 450. The solutions belonging to the Pareto set retained by the filter are presented in Figure 2 and Table I.

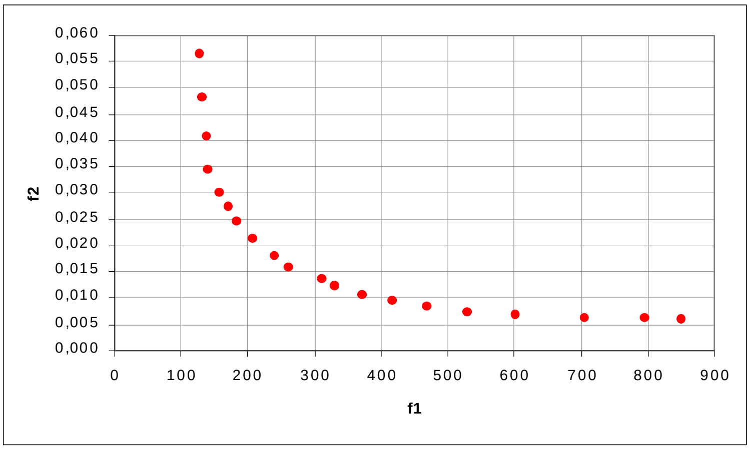

Figure 2: Points in the filter in generation 500.

x1	x2	x3	x4	fo1	fo2
63,60	40,01	0,90	0,90	127,60	0,0565
68,05	40,34	0,90	0,90	132,20	0,0482
72,35	41,26	0,91	0,91	139,00	0,0409
79,12	40,05	0,90	0,90	141,70	0,0345
78,32	45,70	0,92	0,96	158,10	0,0301
79,65	23,03	0,96	2,15	171,80	0,0273
78,91	40,09	0,92	1,41	183,00	0,0245
78,89	43,89	1,00	1,51	208,70	0,0213
78,56	34,83	0,96	2,42	240,00	0,0181
79,66	46,58	1,02	1,99	262,70	0,0158
77,95	42,21	1,04	2,80	311,50	0,0136
78,62	45,11	0,93	2,92	331,40	0,0123
79,79	40,89	0,93	3,72	372,00	0,0106
79,92	40,04	1,01	4,30	416,70	0,0095
79,99	47,49	1,29	3,97	470,50	0,0085
79,58	50,00	1,15	4,49	529,80	0,0075
80,00	48,67	1,66	4,99	602,30	0,0068
79,59	50,00	2,95	5,00	705,00	0,0064
79,72	50,00	4,42	4,87	796,60	0,0062
80,00	50,00	5,00	5,00	850,00	0,0059

Table I: Values of the design variables and the objective functions for the points in Figure 2.

3.2. Example 2: Welded Beam Design

The problem consists of a beam submitted to a force F in its extremity and that needs to be welded to another structural component satisfying stability conditions and project limitations. The four design variables – weld thickness (*h*); length of weld (*l*); width of the beam (*t*) and thickness of the beam (*b*) – are indicated in the Figure 3.

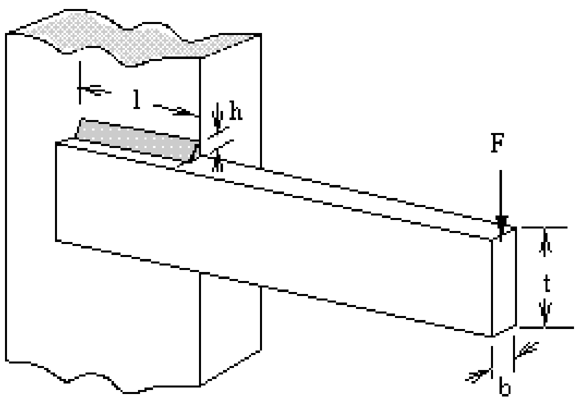

Figure 3: The welded beam [Deb, 1998].

The two objectives for this problem are:

- the cost of the beam;
- the deflection at the end of the beam.

Both objectives are to be minimized but these criteria are again incommensurable and conflicting as lowering the cost usually leads to a higher deflection while, in order to reduce the deflection, it is usually necessary to increase the cost.

There are five strength constraints. The two first constraints make sure that the shear stress and normal stress developed at the support location of the beam are smaller than the allowable shear strength (τ_{max}) and yield strength (σ_{max}) of the material respectively. The third constraint makes sure that the allowable buckling load (along *t* direction) of the beam is greater than the applied load *F*. The fourth constraint is a maximum limit (u_{max}) for the displacement at the end of the beam. The fifth constraint makes sure that the thickness of the beam is not smaller than the weld thickness. There are still other geometric constraints on the decision variables according to the following model:

$$
\begin{aligned}
&\text{Min. } f_1(\mathbf{x}) = 1.10471h^2l + 0.04811tb(14+l) \\
&\text{Min. } f_2(\mathbf{x}) = 2.1952 / (t^3b) \\
&\text{Subject to} \\
&\qquad g_1(\mathbf{x}) \to \tau(\mathbf{x}) - \tau_{max} \le 0 \\
&\qquad g_2(\mathbf{x}) \to \sigma(\mathbf{x}) - \sigma_{max} \le 0 \\
&\qquad g_3(\mathbf{x}) \to F - P_c(\mathbf{x}) \le 0 \\
&\qquad g_4(\mathbf{x}) \to 2.1952 / (t^3b) - u_{max} \le 0 \\
&\qquad g_5(\mathbf{x}) \to h - b \le 0 \\
&\qquad 0.125 \le h, b \le 5.0 \\
&\qquad 0.1 \le l, t \le 10.0
\end{aligned}
\tag{12}
$$

The stress and buckling terms are given as follows [Deb, 1998]:

$$\tau(x) = \sqrt{\tau'^2 + \tau''^2 + l\tau'\tau'' / \sqrt{0.25\left[l^2 + (h+t)^2\right]}} \tag{13}$$

$$\tau' = \frac{6000}{\sqrt{2}hl} \tag{14}$$

$$\tau'' = \frac{6000(14+0.5l)\sqrt{0.25\left[l^2+(h+t)^2\right]}}{2\left\{0.707hl\left[l^2/12+0.25(h+t)^2\right]\right\}} \tag{15}$$

$$\sigma(x) = \frac{504000}{t^2b} \tag{16}$$

$$P_c(x) = 64746.022(1-0.0282346t)tb^3 \tag{17}$$

The remaining data for the problem are [Coello, 1998]:

$$
\begin{aligned}
&F = 6000 \text{ lb} && \tau_{max} = 13600 \text{ psi} \\
&E = 30 \text{ x } 10^6 \text{ psi} && \sigma_{max} = 30000 \text{ psi} \\
&G = 12 \text{ x } 10^6 \text{ psi} && u_{max} = 0.25 \text{ in} \\
&L = 14 \text{ in (free length)} &&
\end{aligned}
\tag{18}
$$

The chromosome is 54 bits long and a simple penalty method is applied:

$$f_i(\mathrm{x}) \leftarrow f_i(\mathrm{x}) + c_i \,.\, nviol \tag{19}$$

where c_1=100; c_2=0,01 and *nviol* is the number of violated constraints. The values of the c_i constants make sure that a solution violating one constraint is worst than a feasible one and that a solution violating two constraints is worst than one violating a single constraint and so on.

The population size is 200 and the number of generations is 100. In Figure 4, the feasible solutions in the initial generation and in generation 100 are shown. The efficiency of the new operators is indicated in Figure 5 which displays the solutions in the filter in generation 100, with (right) and without (left) the use of these operators respectively.

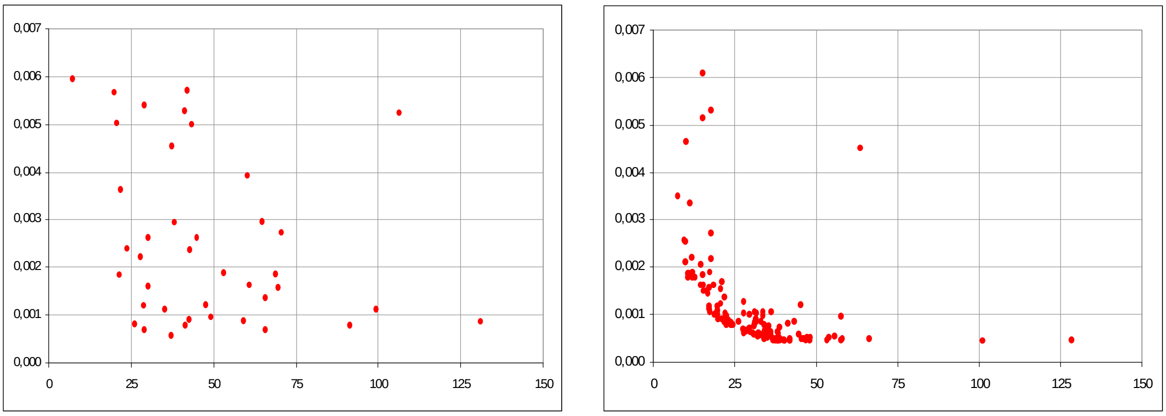

Figure 4: Feasible solutions in the initial generation and in generation 100, respectively.

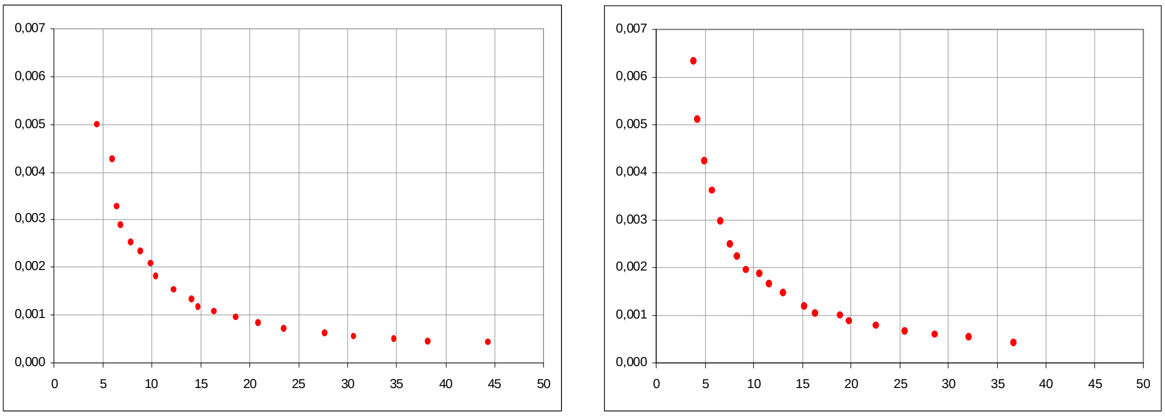

Figure 5: Solutions in the filter in generation 100: without (left) and with (right) the use of the addition and single-objective operators respectively.

The results can be improved if we increase the number of additional generations that act only on the filter to 400, as it can be observed in Figure 6 and Table II.

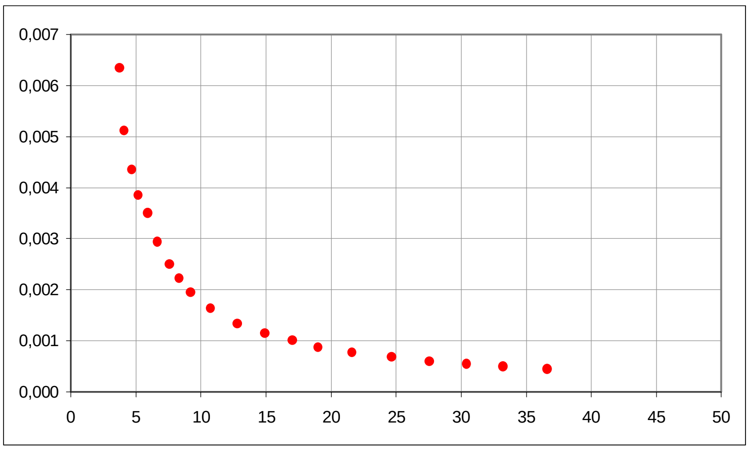

Figure 6: Solutions in the filter in generation 500 (100 + 400).

h	l	t	b	fo1	fo2
0,393	2,888	9,304	0,430	3,742	0,00634
0,317	3,662	9,926	0,439	4,112	0,00511
0,396	3,091	9,981	0,508	4,707	0,00434
0,393	2,927	9,933	0,582	5,209	0,00385
0,413	3,004	9,810	0,666	5,909	0,00349
0,393	2,898	9,926	0,765	6,672	0,00293
0,454	2,471	9,948	0,893	7,600	0,00250
0,739	1,498	9,937	1,005	8,348	0,00223
0,798	1,246	9,966	1,140	9,213	0,00195
1,182	0,787	10,000	1,343	10,770	0,00163
1,136	0,782	10,000	1,648	12,830	0,00133
1,184	0,748	9,942	1,953	14,930	0,00114
1,182	0,903	10,000	2,181	17,030	0,00101
1,182	0,753	10,000	2,519	19,040	0,00087
0,870	1,057	10,000	2,867	21,650	0,00077
1,184	0,787	9,922	3,324	24,680	0,00068
1,182	0,754	9,923	3,753	27,590	0,00060
1,334	0,750	10,000	4,086	30,470	0,00054
1,182	0,748	10,000	4,524	33,250	0,00049
1,182	0,748	10,000	5,000	36,630	0,00044

Table II: Values of the design variables and the objective functions for the points in Figure 6.

4. CONCLUSIONS

The proposed algorithm can find Pareto-optimal solutions in a single run, even without the introduction of sensitive parameters for the resolution of the problem. This shows that a good approximation of the Pareto set can be achieved in practice, improving the understanding and facilitating the solution of optimization problems and, as a consequence, the task of the designers.

No niche strategy or sophisticated penalty method were used, highlighting the effectiveness of the proposed algorithm.

Most part of the merit for the success of this optimization procedure is attributed to the new operators: exclusion, addition and single-objective operator. This last operator allows, in case of need, that the presented algorithm be used in the optimization of a single objective.

5. REFERENCES

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